ON THE CALCULATION OF CHARGE, ELECTROSTATIC POTENTIAL AND CAPACITANCE IN GENERALIZED FINITE SAW STRUCTURES

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Abstract: We present a method for the calculation of charge, electrostatic potential and capacitance for arbitrary, finite SAW-structures. Our method is applicable for one and two dimensional SAW-structures without any restrictions on the finger geometry and allocation. Analytical expressions for the charge and potential distribution are derived by using the method of moments and the relation between charge and potential spectral components. A semi-numerical evaluation procedure with automatic error estimation has been developed. However, computation times are fairly small (several seconds on a medium scale computer), so that our technique is feasible for actual development-oriented SAW design. As a particular result the influence of the end-fingers of SAW transducers on the driving function for surface waves are discussed.

1. Introduction

To our knowledge all previously published techniques for the analysis of charge distribution on the fingers of surface acoustic wave interdigital transducers (SAW-IDT) rely on stringent assumptions about the geometry and driving potentials of the fingers.
Under the assumption of infinite periodicity of the fingers Datta et. al [1] have derived an analytical expression for the element factor, which is proportional to the Fourier transform of the spatial charge distribution if only one finger is activated. Using this fundamental function the response of an infinite periodic transducer can be calculated. However, neither the end-effects in a finite transducer with periodic finger allocation nor the effect of the aperiodicity can be accounted for in this way.
Iterative solutions [2] are time consuming. Predetermined functions for charge-potential-interrelation [3] or for charge distribution with fitting parameters [4] are not accurate enough in general cases.
Using Fourier transform and moment methods Ristic et.al [5] could show that for one-dimensional representation of SAW-IDTs with geometrical symmetry and electrical antisymmetry conditions it is possible to give closed-form expressions for the elements of a charge-potential-interrelation-matrix (CPIM).
The problem of determining charge, capacitance and field distribution thereby reduces to inversion of the (CPIM).
Reformulating and extending the above solution procedure we show that, without imposing any kind of restrictions on electrical characteristics and geometry it is possible to give closed-form expressions for the (CPIM)-elements for one- and two-dimensional representations of SAW-IDTs including floating fingers if necessary.

2. One-Dimensional case

The geometry of interest is shown in Fig.1:

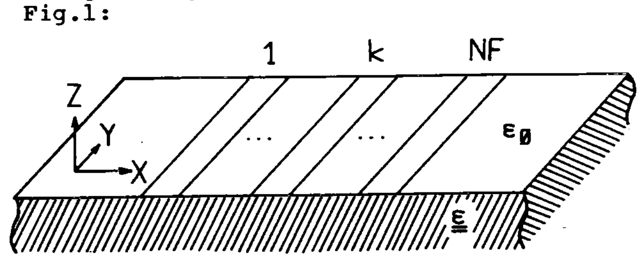

Fig.1 One-dimensional representation of SAW-IDT

There are NF metallic fingers deposited on the surface of a semi-infinite anisotropic dielectric.
Fig.2 shows the j-th finger with a nonequidistant discretisation. The shaded regions are the i-th and $(j\cdot M-i+[(j-1)\cdot M+1])$-th substrips of the j-th finger, with

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$$(j-1)\cdot M \leq i \leq j\cdot M$$

and

M...number of substrips of a finger

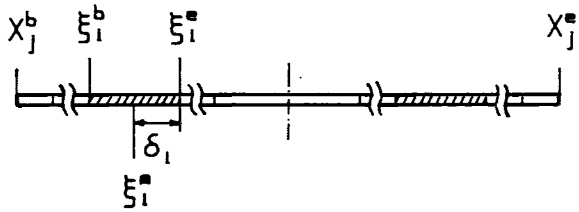

Fig.2 j-th finger

ξ_i^m and $2\delta_i$ are the midpoint-coordinate and the width of the i-th substrip.

1.1 Solution procedure

* Derive the relation between the charge and potential spectral components (the Green function in the wave-number space) for the geometry of interest (a semi-infinite anisotropic dielectric in our case)
  As is well known [5] the relation (1) yields:

$$\bar{\varphi}(K_X) = \bar{G}(K_X)\cdot\bar{\rho}_S(K_X) \qquad \ldots(1)$$

where

$$\bar{G}(K_X) = \frac{1}{\varepsilon_0(1+\alpha)\,|K_X|}$$

α...anisotropy factor.

The bar denotes Fourier transformation.

* Approximate the spatial charge distribution by a sum of stepfunctions.
* Take the Fourier transform of the resulting function:

$$\bar{\rho}(k_x) =$$

$$= \rho_0 \sum \sigma_i\delta_i \exp(jk_x\xi_i^m)\cdot \mathrm{SINC}(k_x\delta_i)$$

with $\quad i = 1..NT \qquad \ldots(2)$

ρ_0...normalization factor
σ_i...unknown charge value on i-th substrip (Fig.2)
$\sigma_i\delta_i$...charge integral on i-th substrip
NF...number of fingers
M...number of subdivisions of one finger (principially,with our formalism this number may vary from finger to finger)
NT...total number of subdivisions (NT=NF·M)

* Insert (2) in (1)
* Calculate the inverse Fourier transform of the resulting function which gives an expression for $\varphi(x)$ as a function of $\sigma_i\delta_i$
* Relate the potential $\varphi(x)$ to that of the NF-th finger
* A point matching procedure of the potential $\varphi(x)$ gives:

$$\varphi(\xi_j^m)-\varphi(\xi_{NT}^m) = \varrho \sum \sigma_i\delta_i A(i,j)$$

ϱ ...proportionality factor $\qquad \ldots(3)$

* Formulate the charge neutrality condition on the whole transducer:

$$\sum A(NT,i)\cdot\sigma_I\delta_I = 0 \qquad \ldots(4)$$

with

$$A(NT,i) = 1 \qquad i=1..NT$$

The coefficients given by (3) and (4) are the elements of (CPIM).
(3) and (4) are NT equations in NT unknowns $\sigma_i\delta_i$.
The capacitance of the j-th finger can be written by:

$$C_j = C_0 \int |\sigma(x)|\,dx$$
$$(\text{j-th FINGER})$$

1.3. Error analysis

Fig.3 shows the i-th substrip of the j-th finger.

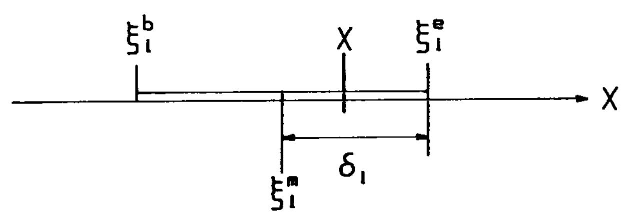

Fig.3 i-th substrip of j-th finger

Now we replace the stepfunction approximation of the charge density on the i-th substrip by a linear function:

$$\sigma(x) = \sigma(\xi_i^m)+(x-\sigma_i^m)\cdot\sigma'(\xi_i^m) \qquad \ldots(5)$$

The same procedure as before leads to the (NT-1) equations:

$$\varphi(\xi_j^m)-\varphi(\xi_{NT}^m) = \sum \sigma_i A(i,j)+ET(j) \qquad \ldots(6)$$

Et(j) are analytical solutions of integrals, and represent the error terms.
These terms are of the order of some mV if the driving potentials of the fingers are some volts. Therefore a non-equidistant discretisation of the fingers with a stepfunction approximation is accurate enough for most cases of interest in practice.

1.3. Applications

The theory developed here can be applied to a variety of applications including:

* halfspace structures
* plate structures
* striplines
* multilayer structures

It is worth noting that complex structures lead to difficult Green functions in wave-number space. The elements of the (CPIM) are integrals, their complexity depending on the Green function. Therefore it is not always possible to give closed-form solutions for these integrals.

2. Floating fingers

To simplify the discussion we consider a 3-finger transducer with one floating finger. All fingers are divided into 2 subsections Fig.4:

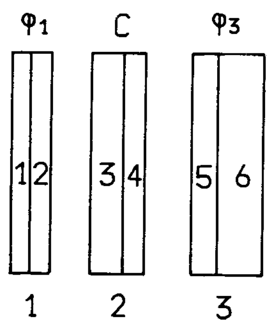

Fig.4 One floating finger neighboured by two active fingers

Boundary condition for a floating finger:
The potential of a floating finger which is not known a priori, has to be determined in a way such that the whole charge on it is zero.
This charge neutrality condition for the floating finger in Fg.4 gives:

$$-\sigma_3\delta_3-\sigma_4\delta_4 = \emptyset \qquad \ldots (7)$$

One can easily show that the matrix in (8) represents the (CPIM) of Fig.4.

It should be noted that the unknown potential c and the charge integrals $\sigma_i\delta_i$ can be calculated simultaneously.
According to the matrix equation (8) we conclude that the inclusion of floating fingers requires only to increase the rank of (CPIM) by the number of floating fingers in a very simple manner:
The additional rows and columns describing floating fingers are diagonally symmetric and have elements that are -1 or 0.

$$\begin{pmatrix} A_{1.1} & . & . & . & . & A_{1.6} & \emptyset \\ . & . & . & . & . & . & \emptyset \\ . & . & . & . & . & . & -1 \\ . & . & . & . & . & . & -1 \\ A_{5.1} & . & . & . & . & A_{5.6} & \emptyset \\ 1 & 1 & 1 & 1 & 1 & 1 & \emptyset \\ \emptyset & \emptyset & -1 & -1 & \emptyset & \emptyset & \emptyset \end{pmatrix} \begin{pmatrix} \sigma_1\delta_1 \\ \sigma_2\delta_2 \\ \sigma_3\delta_3 \\ \sigma_4\delta_4 \\ \sigma_5\delta_5 \\ \sigma_6\delta_6 \\ C \end{pmatrix} = \begin{pmatrix} \varphi_1-\varphi_3 \\ \varphi_1-\varphi_3 \\ -\varphi_3 \\ -\varphi_3 \\ \varphi_3-\varphi_3 \\ \emptyset \\ \emptyset \end{pmatrix} \qquad \ldots (8)$$

3. Two-Dimensional case

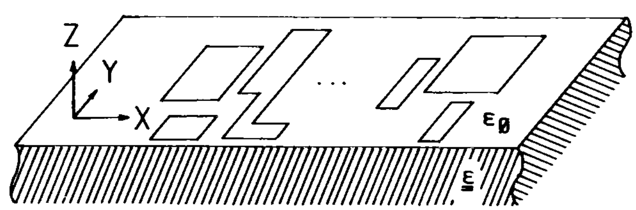

Fig.5 Two-dimensional representation of SAW-IDT

As in the 1-Dim. case we have to find the 2-Dim. Green function $\bar{G}(k_x,k_y)$ in the wave-number domain. As explained for the 1-Dim. case it relates the spectral components of the potential to the charge:

$$\bar{\varphi}(k_X,k_Y) = \bar{G}(k_X,k_Y)\cdot\bar{\rho}_s(k_X,k_Y) \qquad \ldots (9)$$

One can show that $\bar{G}(k_x,k_y)$ has the form:

$$\bar{G}(k_X,k_Y) = \frac{1}{\varepsilon_0\left[\sqrt{k_X^2+k_Y^2}+\sqrt{\alpha_1 k_X^2+\alpha_2 k_Y^2+2\alpha_{12}k_X k_Y}\,\right]} \qquad \ldots(10)$$

$\alpha_1, \alpha_2, \alpha_{12}$...anisotropy factors.

The same solution procedure as in the 1-Dim. case furnishes the elements of (CPIM), which are sums of integrals. We can show that these integrals also can be calculated analytically. Due to space limitations this cannot be presented here.

4. Results

Now we present some of the results we obtained using the preceding theory in the one-dimensional case.
Fig.6 shows a transducer with one active finger and i grounded fingers to the left and j grounded fingers to the right. This is denoted by (i-1-j).

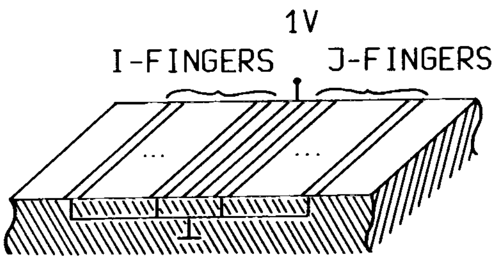

Fig.6 Transducer with one active finger

Fig.7 shows the definition of the metallisation ratio in a transducer with a finite number of fingers deposited equidistantly on the surface.

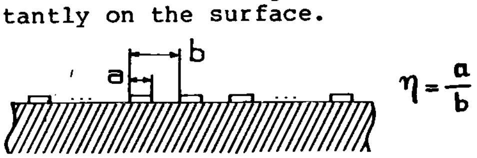

Fig.7 Definition of metallization ratio

Fig.8 shows a finger devided into M-subsections.

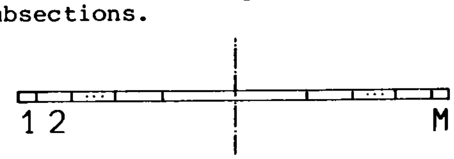

Fig.8 j-th finger divided into M substrips

With these definitions of (i-1-j); η and M we can now discuss the curves depicted in figures 9 and 10 in greater detail. The curves represent the excitation functions $\bar{\rho}(k)\cdot k^{1/2}$ for the first eight harmonics in dependence of the wave-number in a normalized form.
In the case of infinite periodicity, these excitation functions degenerate to the well known element-factor. The curve at the top of Fig.9 corresponds to the element-factor of an infinitely periodic transducer as published in [1]. This means that the infinite periodicity can be simulated using only few fingers. Fig.9 shows that the first finger (bottom curve in the figure) yields a spectrum which is completely different from that of other fingers. Fig.10 shows the combined influence of left and right end-fingers.

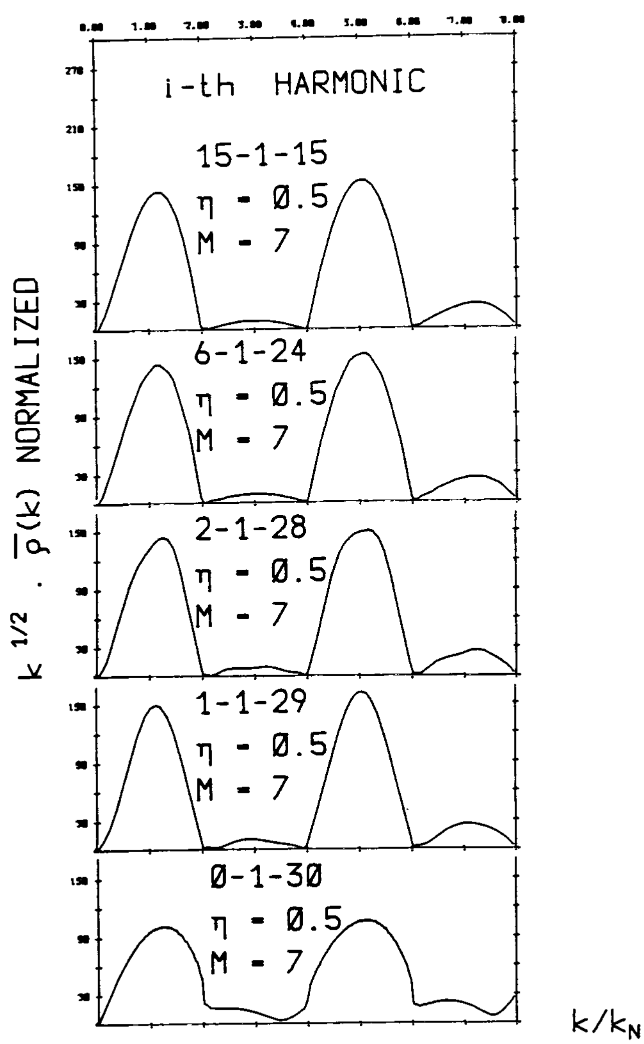

Fig.9 Influence of left end-fingers

Conclusion

We have presented a method for calculation of the charge density on the fingers of a generalized finite SAW-IDT. The elements of the charge-potential-interrelation matrix (CPIM) have been calculated in a closed-form, so that the problem of determining the charge density on the fingers essentially reduces to inversion of the (CPIM). A simple extension of (CPIM) allows the inclusion of floating fingers. It has been claimed that the elements of (CPIM) in the 2-Dim. case can also be calculated analytically.

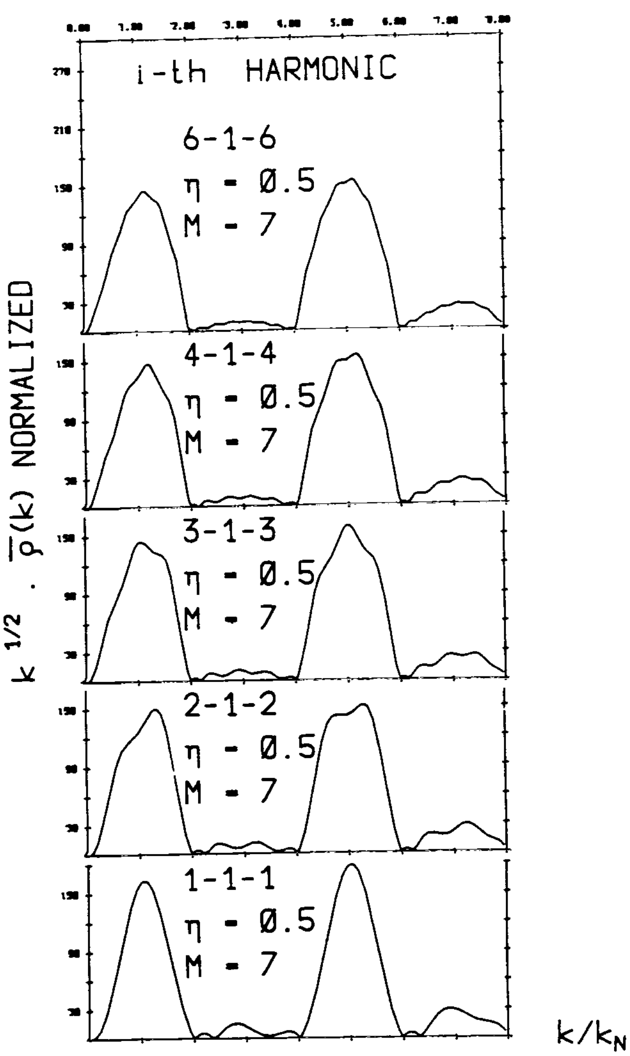

Fig.10 Influence of left and right end-fingers

Acknowledgement

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