A SINGULAR PERTURBATION APPROACH FOR THE ANALYSIS OF THE FUNDAMENTAL SEMICONDUCTOR EQUATIONS

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The purpose of our work was the analysis of the two dimensional steady state semiconductor equations and of the numerical method quoted in /1/ using a singular perturbation approach. The one dimensional model has been analysed in the framework of singular perturbations in /2/ and /3/. After scaling the carrier densities and the doping to order O(1) the equations can be brought into the form :

$$\lambda^2 \operatorname{div}(\operatorname{grad}\psi) = n - p - C$$
$$\operatorname{div}(D_n \operatorname{grad} n - \mu_n n.\operatorname{grad}\psi) = R(n,p,\lambda)$$
$$\operatorname{div}(D_p \operatorname{grad} p + \mu_p p.\operatorname{grad}\psi) = R(n,p,\lambda)$$

Here λ acts as the perturbation parameter which is proportional to the Debeye - length. It follows from the theory of singular perturbations that in subdomains the solution is approximated up to order $O(\lambda)$ by the solution of the reduced problem (where λ is set to zero). At the boundaries between two such subdomains there occur layers, i.e. small regions where the derivatives of the solution grow as $1/\lambda$ when λ approaches zero. Equations for the limits of the reduced solution, as the independent variables tend to such a boundary from both sides, are derived. It emerges that the jump of the reduced solution for n and p depends exponentially on the jump of ψ. Since the solution of the full problem exhibits a strongly different behaviour inside and outside the layers it is necessary to discretize the problem on a nonuniform grid. For Poisson's equation the usual first order discretization of the Laplacian is used. For the continuity equations a method is used whose stability properties are independent of the direction of $\operatorname{grad}\psi$. It is investigated how the mesh has to be chosen to obtain a good approximation in compact subsets of the various subdomains. Away from the layers the difference equations approximate the reduced equations up to order $O(\lambda)$. There it is sufficient to control the usual estimate of the truncation errors. Furthermore it is necessary that the jumps of the reduced solution for the carrier densities inside the layers caused by the unboundedness of $\operatorname{grad}\psi$ as λ approaches zero are approximated as accurately as possible. Thus the mesh has to be chosen in a way such that the equation for the onesided limits of the reduced solution at the layers is solved automatically by difference scheme. Criteria for this property are given which can be used to control the mesh.

/1/ Franz A.F.,Franz G.A.,et. al., **"Finite Boxes - A Generalization of the Finite Difference Method utmost suitable for Semiconductor Device Simulation"**, submitted to Conf. on Numerical Simulation of VLSI Devices, Boston 1982

/2/ Markowich P.,Ringhofer C.A.,et al.,**"A singularity perturbed boundary value problem modelling a semiconductor device"**, submitted to SIAM J.Appl.Math., 1982

/3/ Smith D.R., **"On a singularly perturbed boundary value problem arising in the physical theory of semiconductors"**, Report of the Inst.f.Math. TU Munich, 1980