

TWO DIMENSIONAL MOS-TRANSISTOR MODELING

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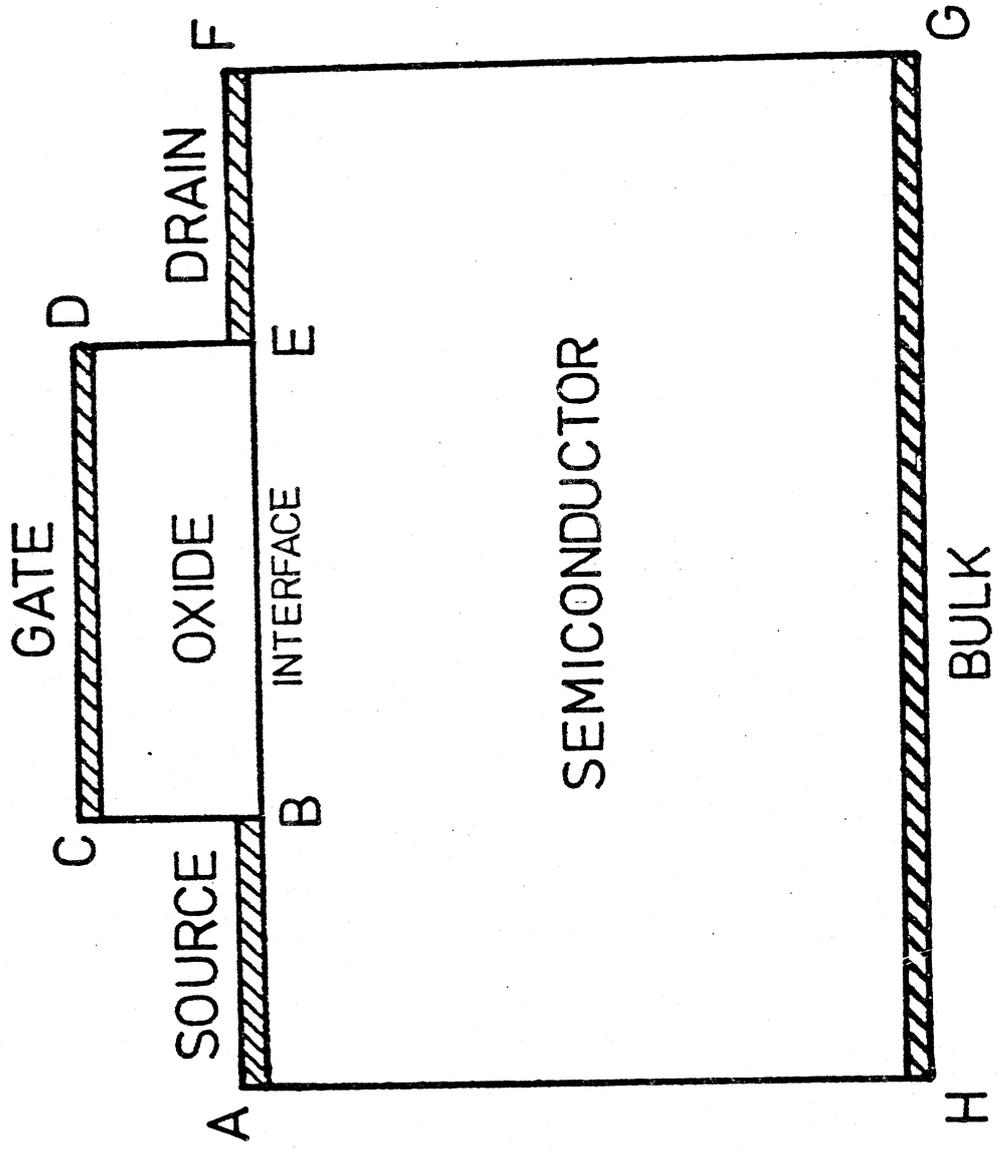
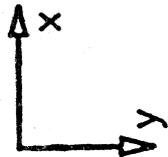
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## THE BASIC SEMICONDUCTOR EQUATIONS

### ● Poisson's equation:

$$\text{div } \epsilon \text{ grad } \psi = -q ( p - n + C )$$

### ● Continuity equations:

$$\text{div } \vec{J}_n = -q ( G - R )$$

$$\text{div } \vec{J}_p = q ( G - R )$$

### ● Current relations:

$$\vec{J}_n = -q ( \mu_n n \text{ grad } \psi - D_n \text{ grad } n )$$

$$\vec{J}_p = -q ( \mu_p p \text{ grad } \psi + D_p \text{ grad } p )$$

### ASSUMPTIONS MADE

- Constant and isotropic permittivity:

$$\epsilon_{\text{si}} = 11.7 \quad \epsilon_{\text{ox}} = 3.9$$

- Total ionization of impurities:

$$C = N_D - N_A = N_D^+ - N_A^-$$

- No degeneracy:

$$n_i = n_i(T) = 3.88 \cdot 10^{16} \cdot T^{1.5} \cdot e^{-7000/T} \quad (\text{cm}^{-3})$$

- No majority carrier current flow (for the non-avalanche case):

$$J_p = 0 \quad (\text{for n-channel devices})$$

$$J_n = 0 \quad (\text{for p-channel devices})$$

- Constant temperature:

$$T = \text{const}, \quad U_T = k \cdot T / q, \quad 250\text{K} \leq T \leq 450\text{K}$$

- Boltzmann statistics:

$$n = n_i \cdot e^{(\psi - \psi_n) / U_T} \quad p = n_i \cdot e^{(\psi_p - \psi) / U_T}$$

- validity of Einstein-Nernst relation:

$$D_n = \mu_n \cdot U_T \quad D_p = \mu_p \cdot U_T$$

## MODELING OF THE DOPING PROFILE

- External one-dimensional doping data.

$$C(x,y) = C((y^2 + \max(x/f, 0)^2)^{1/2})$$

(x .GT. 0) ... oxid mask  
 (x .EQ. 0) ... mask corner  
 (x .LE. 0) ... free surface

- Analytical predeposition:

$$L_d = 2 \cdot \sqrt{D \cdot t}$$

$$C_p(x,y) = 0.5 \cdot N_s \cdot e^{-(y/L_d)^2} \cdot \operatorname{erfc}(x/L_d)$$

- Analytical ion-implantation and diffusion:

$$a = (2 + (L_d/\Delta R_p)^2)^{-1/2}$$

$$K(y) = e^{-(a \cdot (R_p - y)/\Delta R_p)^2} \cdot \operatorname{erfc}(-a \cdot ((R_p/\Delta R_p) + \sqrt{2} \cdot y/L_d))$$

$$C_i(x,y) = (a/(4 \cdot \Delta R_p \cdot \sqrt{\pi})) \cdot \text{Dose} \cdot (K(y) + K(-y)) \cdot \operatorname{erfc}(x/L_d)$$

- Diffusion constants:

$$D = D_0 \cdot e^{T_a/T}$$

Element	$D_0 / (\text{cm}^2 \text{s}^{-1})$	$T_a / (\text{K})$
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B	0.5554	$-3.975 \cdot 10^4$
P	3.85	$-4.247 \cdot 10^4$
SB	12.9	$-4.619 \cdot 10^4$
A	24.	$-4.735 \cdot 10^4$

● Projected range parameters:

$$R_p = \sum_{i=1}^n a_i \cdot x^i \quad (\mu\text{m})$$

$$\Delta R_p = \sum_{i=1}^n b_i \cdot x^i \quad (\mu\text{m})$$

Coefficients for  $R_p$  in silicon:

Element	B	P	SB	A
$a_1$	$3.338 \cdot 10^{-3}$	$1.259 \cdot 10^{-3}$	$8.887 \cdot 10^{-4}$	$9.818 \cdot 10^{-4}$
$a_2$	$-3.308 \cdot 10^{-6}$	$-2.743 \cdot 10^{-7}$	$-1.013 \cdot 10^{-5}$	$-1.022 \cdot 10^{-5}$
$a_3$		$1.290 \cdot 10^{-9}$	$8.372 \cdot 10^{-8}$	$9.067 \cdot 10^{-8}$
$a_4$			$-3.056 \cdot 10^{-10}$	$-3.442 \cdot 10^{-10}$
$a_5$			$4.028 \cdot 10^{-13}$	$4.608 \cdot 10^{-13}$

Coefficients for  $\Delta R_p$  in silicon:

Element	B	P	SB	A
$b_1$	$1.781 \cdot 10^{-3}$	$6.542 \cdot 10^{-4}$	$2.674 \cdot 10^{-4}$	$3.652 \cdot 10^{-4}$
$b_2$	$-2.086 \cdot 10^{-5}$	$-3.161 \cdot 10^{-6}$	$-2.885 \cdot 10^{-6}$	$-3.820 \cdot 10^{-6}$
$b_3$	$1.403 \cdot 10^{-7}$	$1.371 \cdot 10^{-8}$	$2.311 \cdot 10^{-8}$	$3.235 \cdot 10^{-8}$
$b_4$	$-4.545 \cdot 10^{-10}$	$-2.252 \cdot 10^{-11}$	$-8.310 \cdot 10^{-10}$	$-1.202 \cdot 10^{-10}$
$b_5$	$5.525 \cdot 10^{-13}$		$1.084 \cdot 10^{-13}$	$1.601 \cdot 10^{-13}$

Coefficients for  $R_p$  in silicon-dioxide:

Element	B	P	SB	A
$a_1$	$3.258 \cdot 10^{-3}$	$9.842 \cdot 10^{-4}$	$7.200 \cdot 10^{-4}$	$7.806 \cdot 10^{-4}$
$a_2$	$-2.113 \cdot 10^{-6}$	$-2.240 \cdot 10^{-7}$	$-8.054 \cdot 10^{-6}$	$-7.899 \cdot 10^{-6}$
$a_3$			$6.641 \cdot 10^{-8}$	$7.029 \cdot 10^{-8}$
$a_4$			$-2.422 \cdot 10^{-10}$	$-2.653 \cdot 10^{-10}$
$a_5$			$3.191 \cdot 10^{-13}$	$3.573 \cdot 10^{-13}$

● Implantation through an oxide:

$$R_p = R_{p_{Si}} \cdot (1 - T_{iOx} / R_{p_{Ox}})$$

## MOBILITY MODELING

● Lattice scattering:

$$\mu_L(T) = A \cdot T^{-g} \quad (\text{cm}^2/\text{Vs})$$

$$A_n = 7.12 \cdot 10^8$$

$$g_n = 2.3$$

$$A_p = 1.35 \cdot 10^8$$

$$g_p = 2.2$$

● Lattice and impurity scattering:

$$\mu_{LI}(N, T) = \mu_L(T) \cdot a + \mu_{\min} \cdot (1 - a) \quad (\text{cm}^2/\text{Vs})$$

$$a = \frac{1}{1 + (T/300)^b \cdot (N/N_0)^c}$$

$$N = N_D^+ + N_A^-$$

$$\mu_{\min n} = 55.24$$

$$b_n = -3.8$$

$$c_n = 0.73$$

$$N_{0n} = 1.072 \cdot 10^{17}$$

$$\mu_{\min p} = 49.7$$

$$b_p = -3.7$$

$$c_p = 0.7$$

$$N_{0p} = 1.606 \cdot 10^{17}$$

● Lattice, impurity and surface scattering:

$$\mu_{LIS}(y, E_p, E_t, N, T) = \mu_{LI}(N, T) \cdot \frac{y + y_r}{y + b \cdot y_r} \quad (\text{cm}^2/\text{Vs})$$

$$y_r = y_0 / (1 + E_p / E_{p0})$$

$$b = 2 + E_t / E_{t0}$$

$$E_p = \max(0, (E_x \cdot J_x + E_y \cdot J_y) / (J_x^2 + J_y^2)^{1/2})$$

$$E_t = \max(0, (E_x \cdot J_y - E_y \cdot J_x) \cdot J_x / (J_x^2 + J_y^2))$$

$$y_{0n} = 5 \cdot 10^{-7} \quad y_{0p} = 4 \cdot 10^{-7}$$

$$E_{p0n} = 10^4 \quad E_{p0p} = 8 \cdot 10^3$$

$$E_{t0n} = 1.8 \cdot 10^5 \quad E_{t0p} = 3.8 \cdot 10^5$$

● Lattice, impurity, surface scattering and velocity saturation:

$$\mu_{\text{tot}}(y, E_p, E_t, N, T) = (\mu_{LIS}(\dots))^{B_n + (v_s / E_p)^{B_p}} \cdot 1/B$$

$$v_{sn} = 1.53 \cdot 10^9 \cdot T^{-0.87}$$

$$B_n = -2$$

$$v_{sp} = 1.62 \cdot 10^8 \cdot T^{-0.52}$$

$$B_p = -1$$

## GENERATION/RECOMBINATION MODELING

• Thermal generation/recombination:

$$(G - R)_{th} = \frac{n_i^2 - p \cdot n}{\tau_n (p + n_i) + \tau_p (n + n_i)} \quad (1/\text{cm}^3 \text{s})$$

$$\tau_n = 1 \cdot 10^{-6} \qquad \tau_p = 1 \cdot 10^{-6}$$

• Surface generation/recombination (only in avalanche case):

$$(G - R)_s = \frac{n_i^2 - p \cdot n}{(p + n_i)/s_n + (n + n_i)/s_p} \cdot \delta(y) \quad (1/\text{cm}^3 \text{s})$$

$\delta(y)$  : Dirac-Delta function,  $y=0$  denotes the interface

$$s_n = 100 \qquad s_p = 100$$

• Auger generation/recombination (only in avalanche case):

$$(G - R)_{Aug} = (n_i^2 - p \cdot n) (C_n \cdot n + C_p \cdot p) \quad (1/\text{cm}^3 \text{s})$$

$$C_n = 2.8 \cdot 10^{-31} \qquad C_p = 9.9 \cdot 10^{-32}$$

• Avalanche generation:

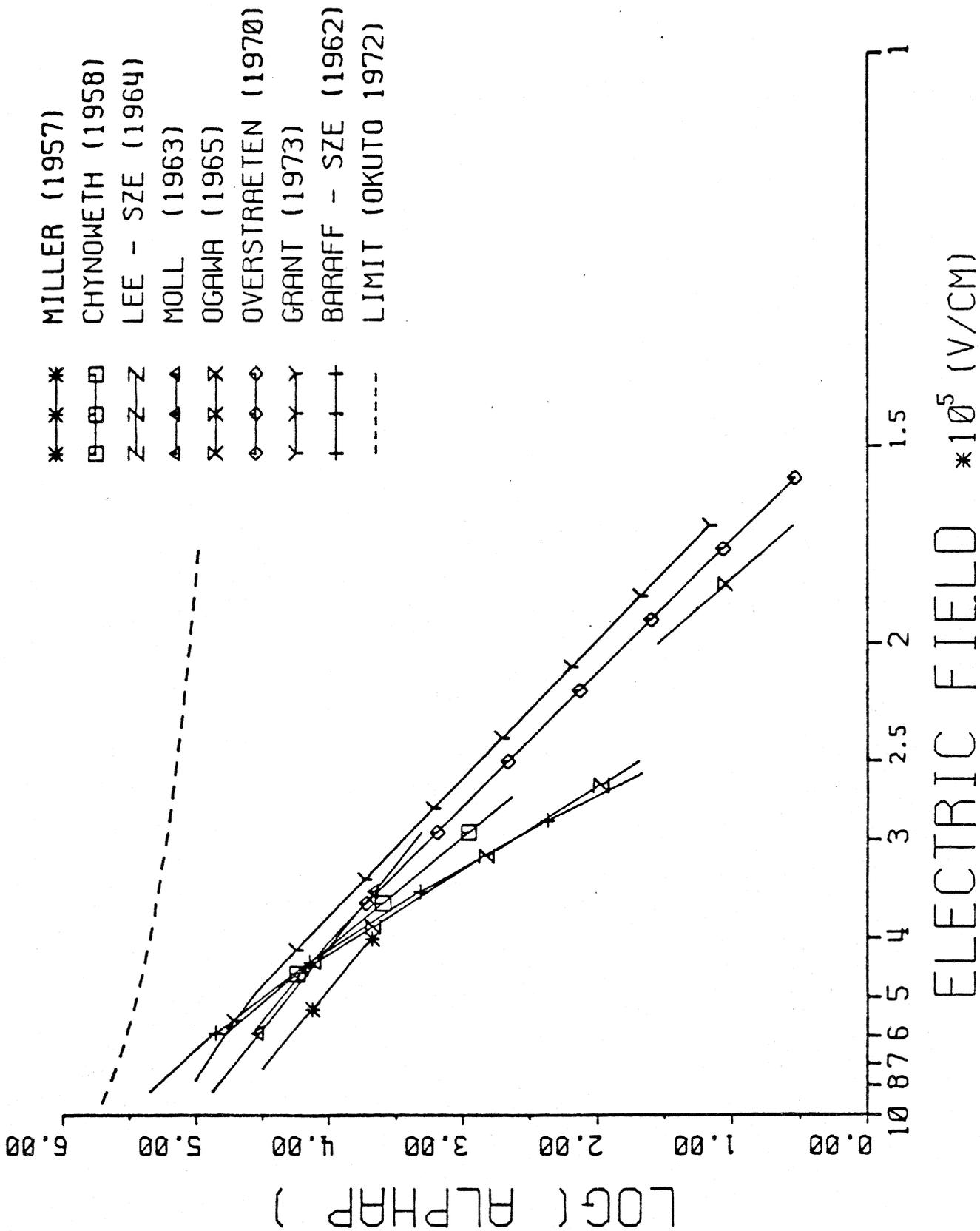
$$G_a = \frac{|\vec{J}_n|}{q} A_n \exp \left( - \frac{B_n |\vec{J}_n|}{\vec{E} \cdot \vec{J}_n} \right) + \frac{|\vec{J}_p|}{q} A_p \exp \left( - \frac{B_p |\vec{J}_p|}{\vec{E} \cdot \vec{J}_p} \right) \quad (1/\text{cm}^3 \text{s})$$

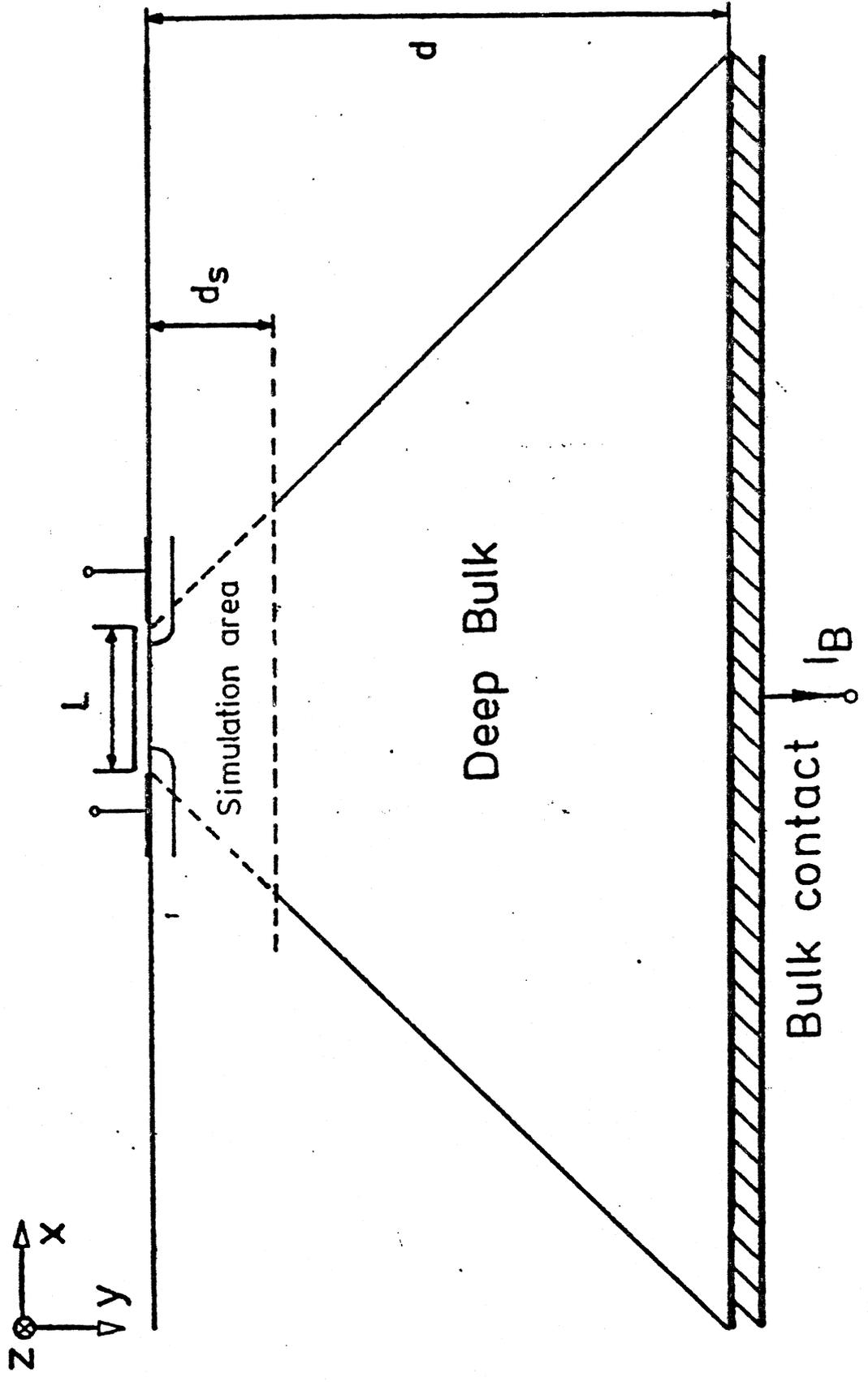
$$A_n = 7 \cdot 10^5 \qquad A_p = 1.588 \cdot 10^6$$
$$B_n = 1.23 \cdot 10^6 \qquad B_p = 2.036 \cdot 10^6$$

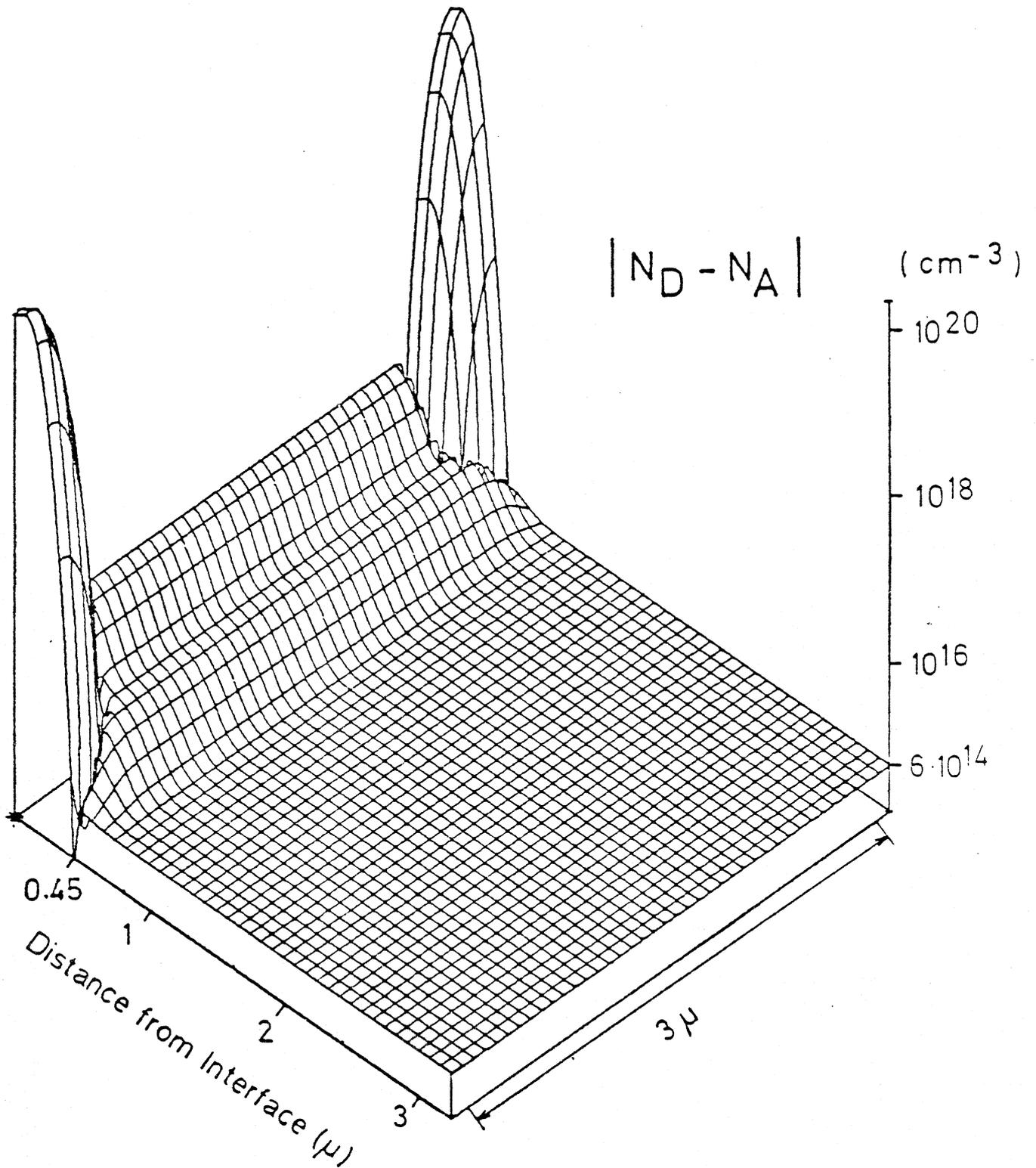
• Total generation/recombination:

$$(G - R)_{tot} = (G - R)_{th} + (G - R)_s + (G - R)_{Aug} + G_a$$

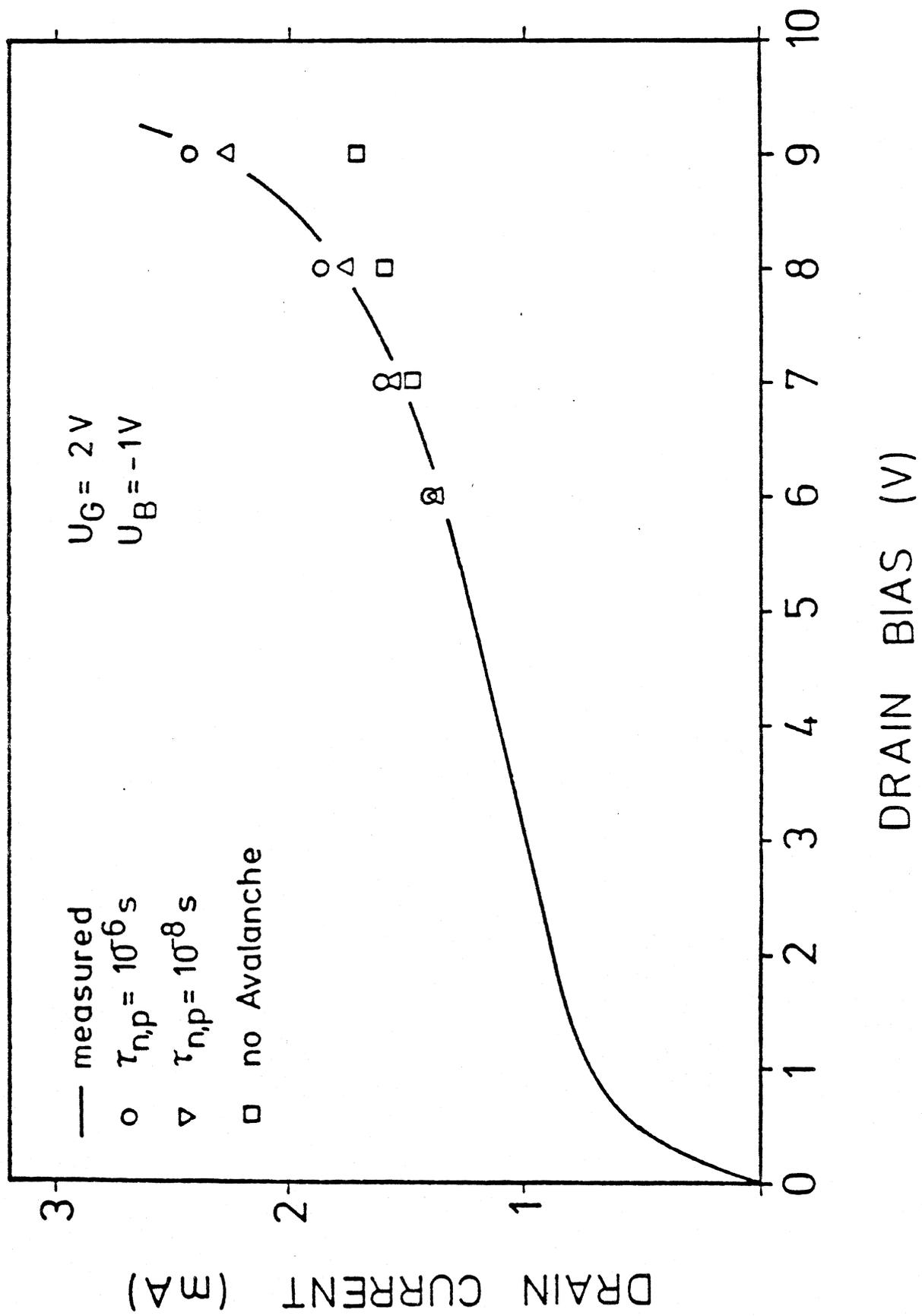


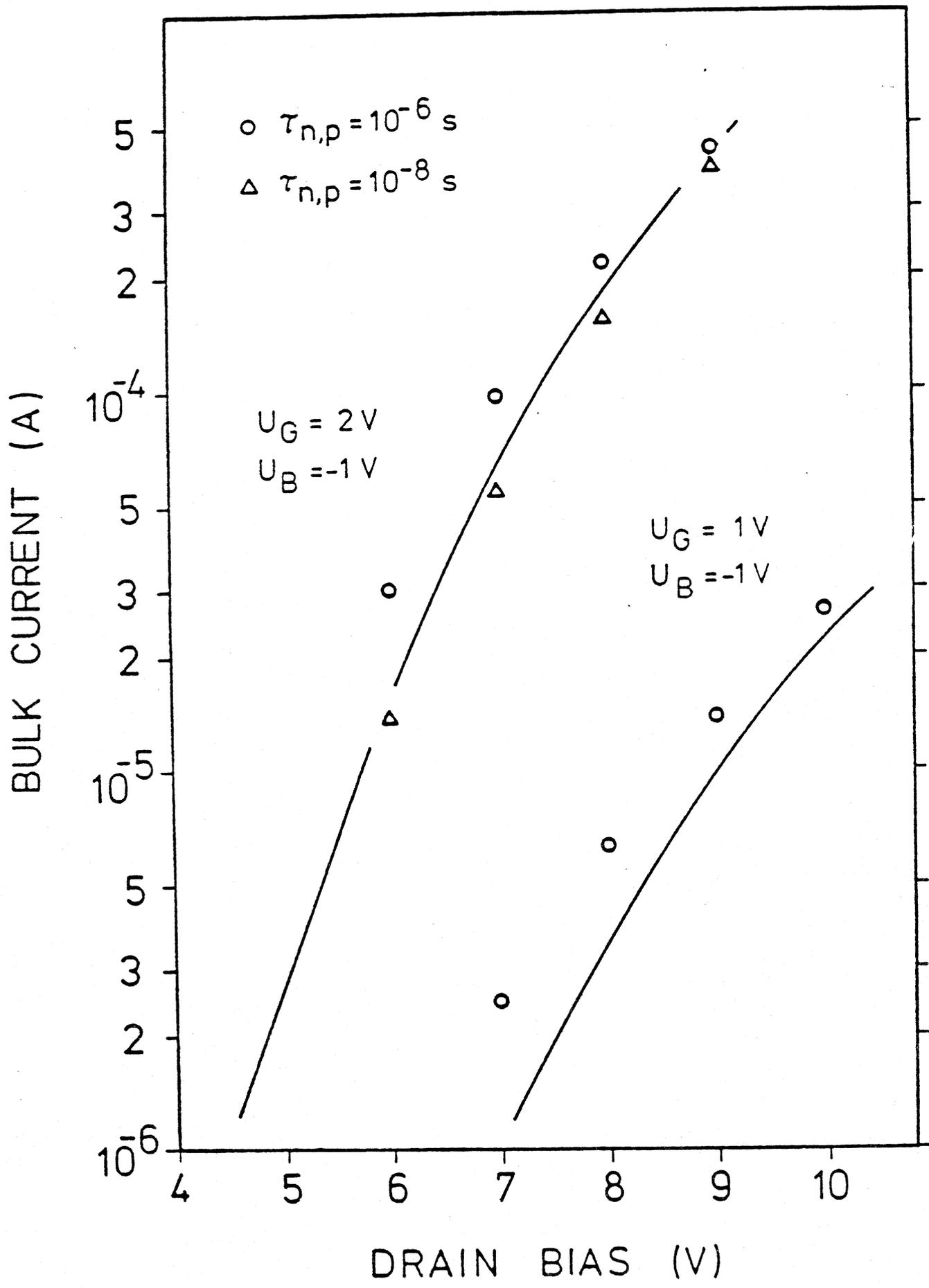


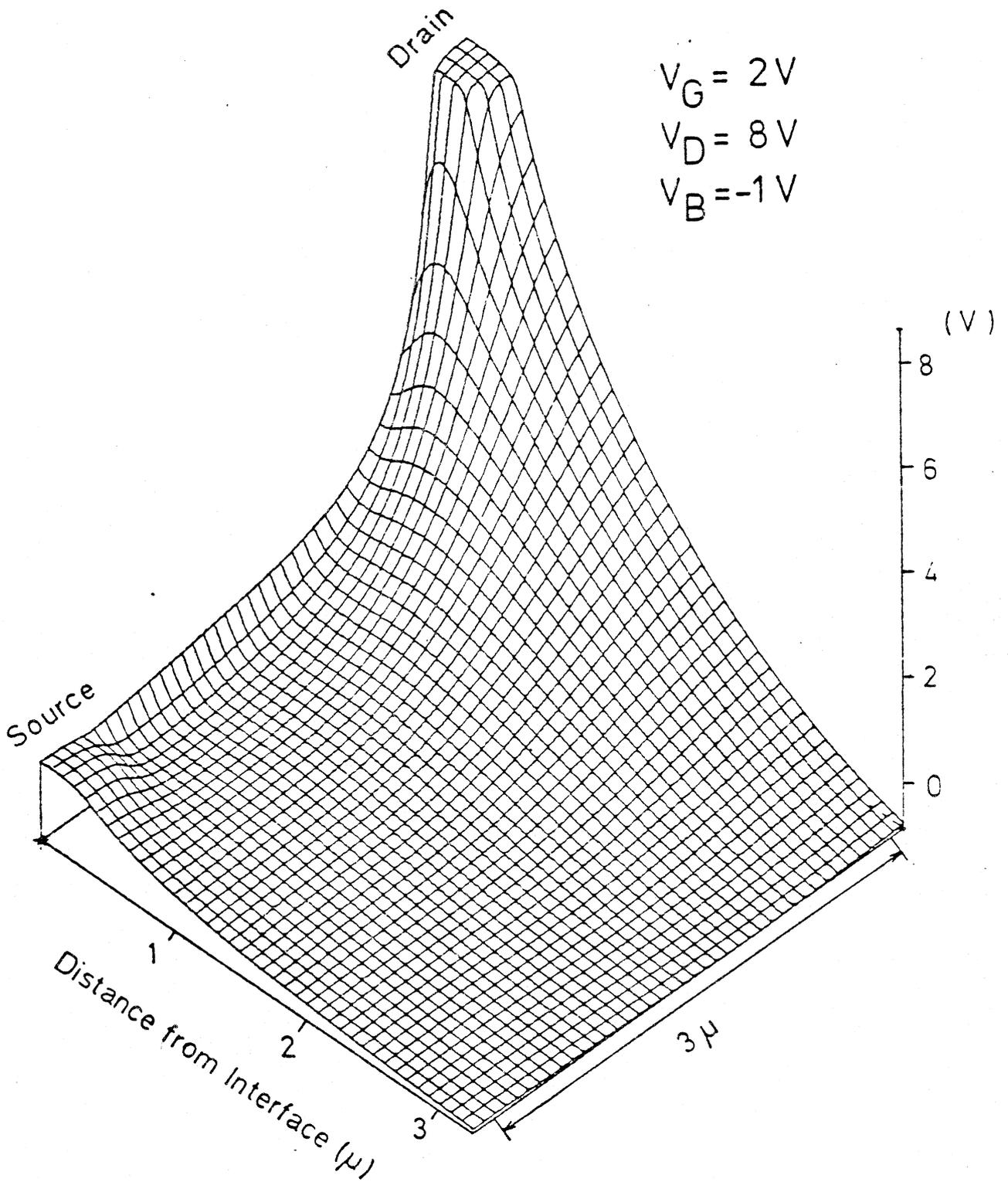




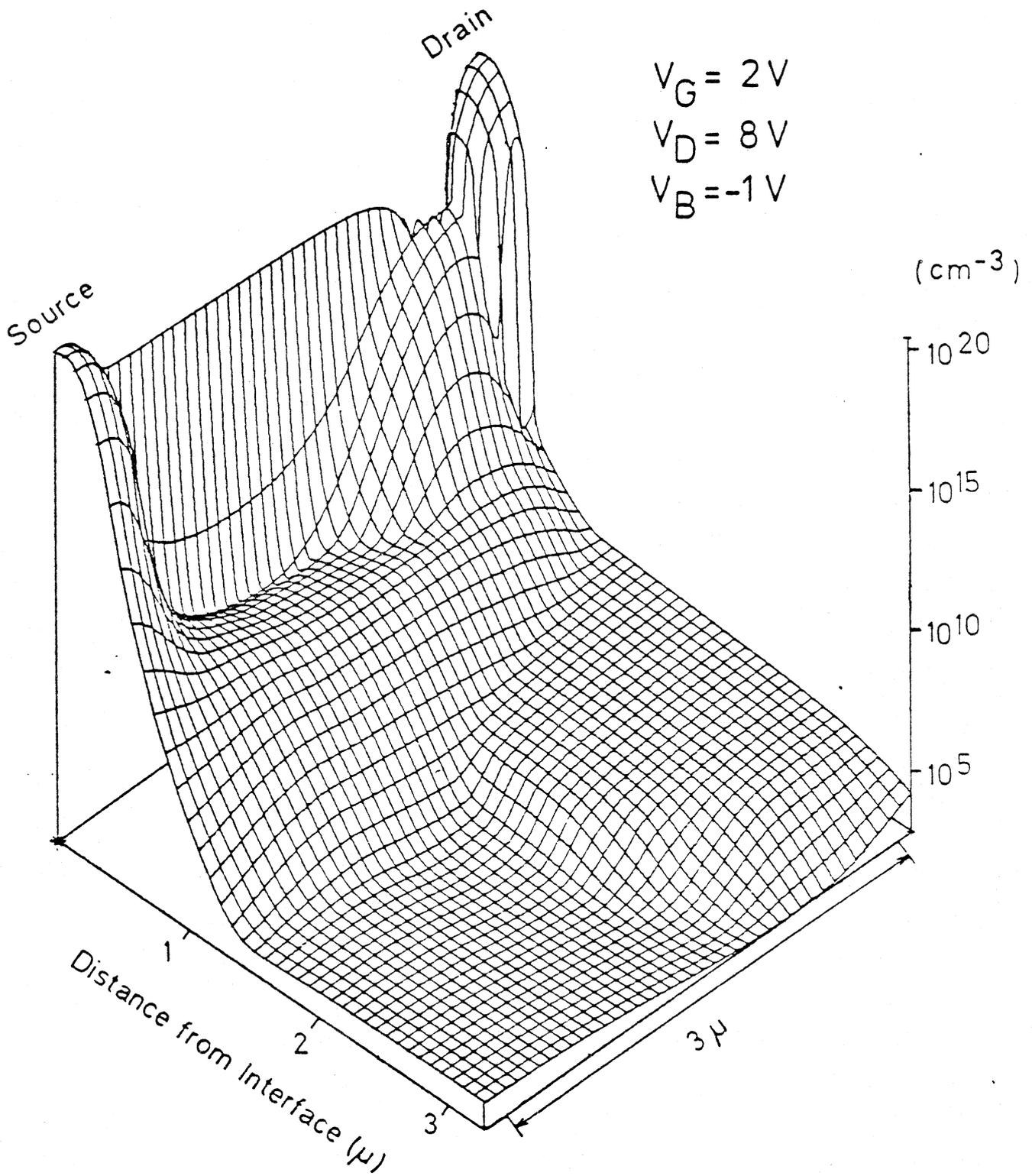
DOPING CONCENTRATION



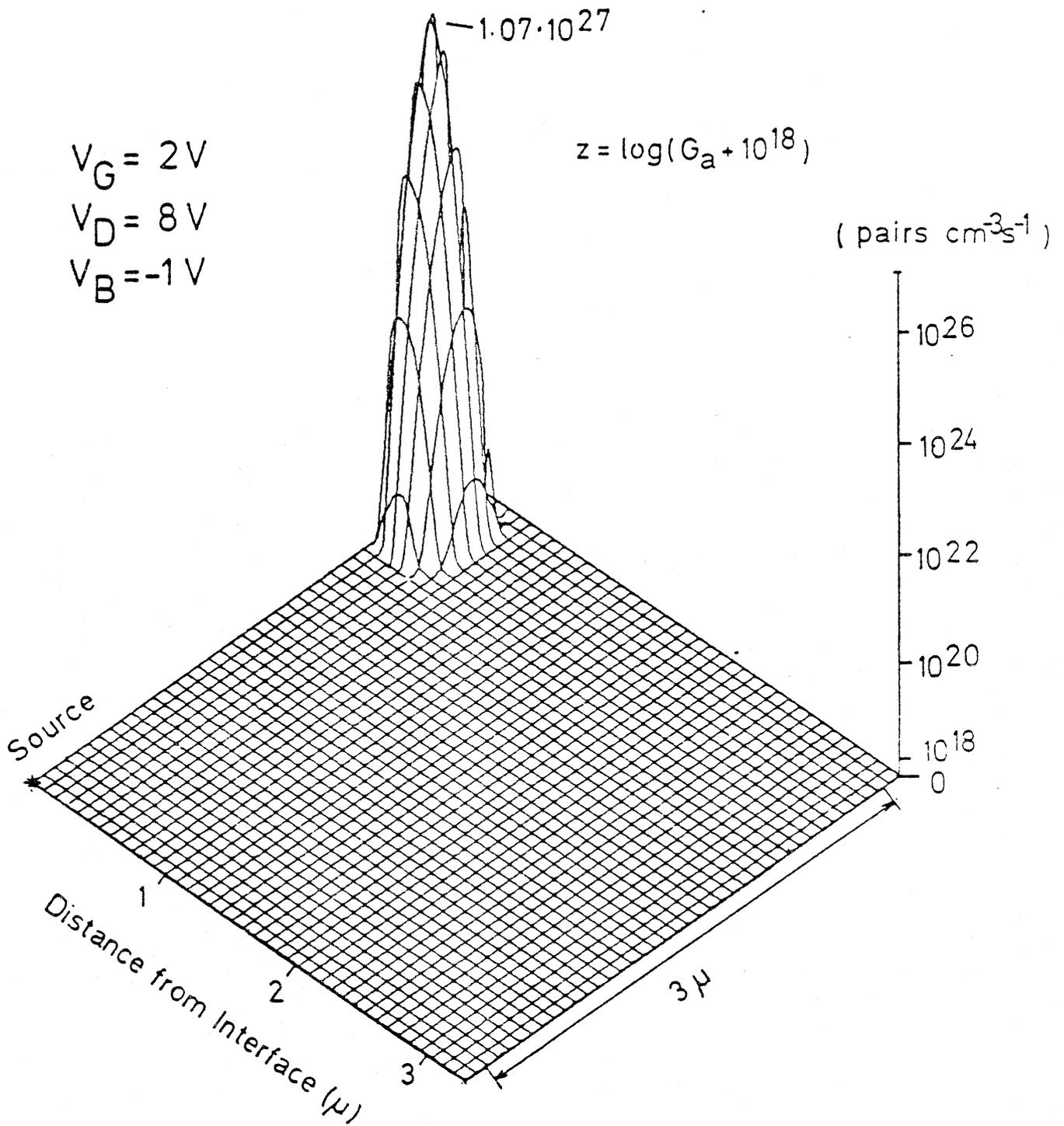




ELECTRICAL POTENTIAL



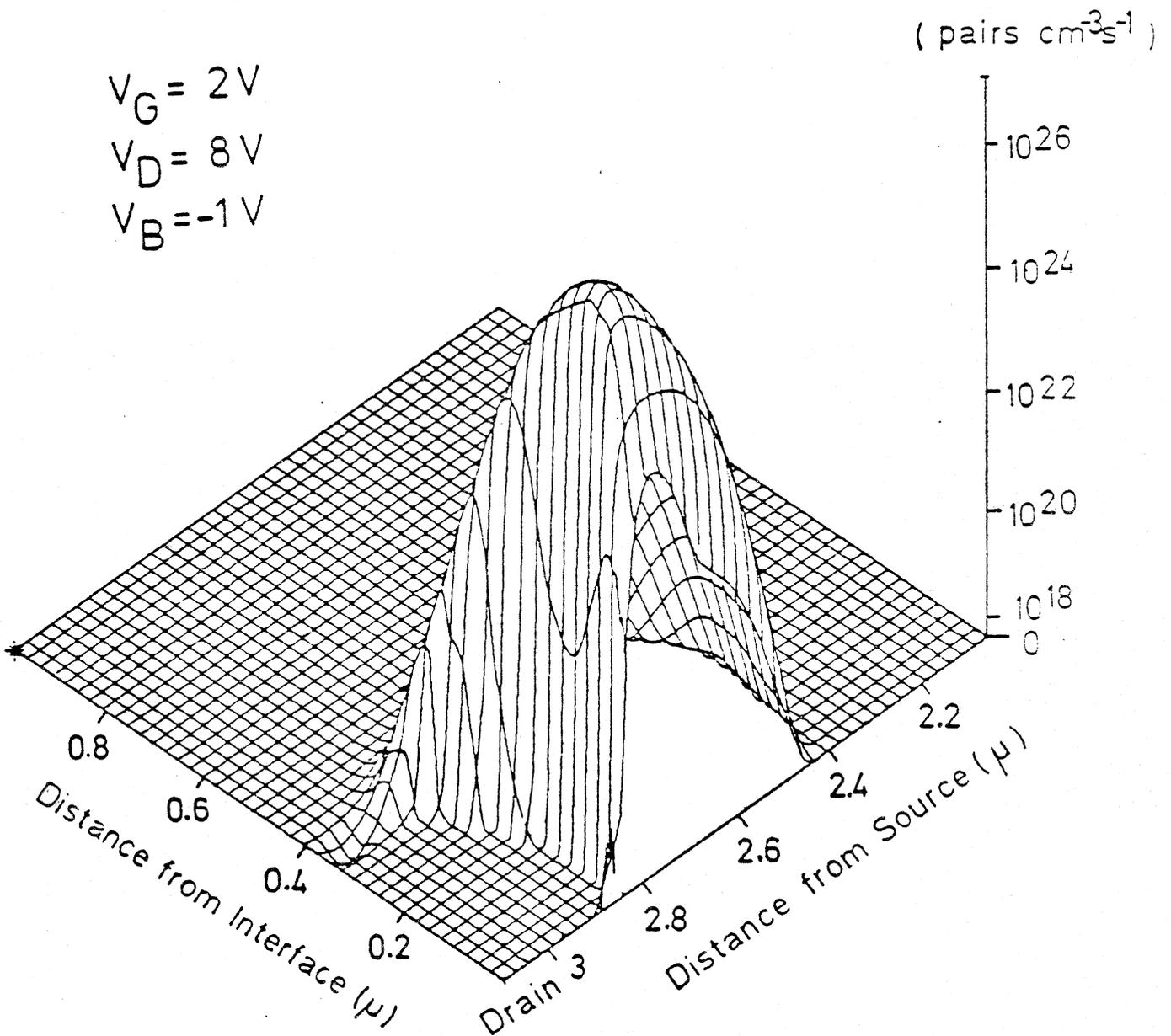
ELECTRON DISTRIBUTION



# AVALANCHE GENERATION

$$z = \log(G_a + 10^{18})$$

$$V_G = 2V$$
$$V_D = 8V$$
$$V_B = -1V$$



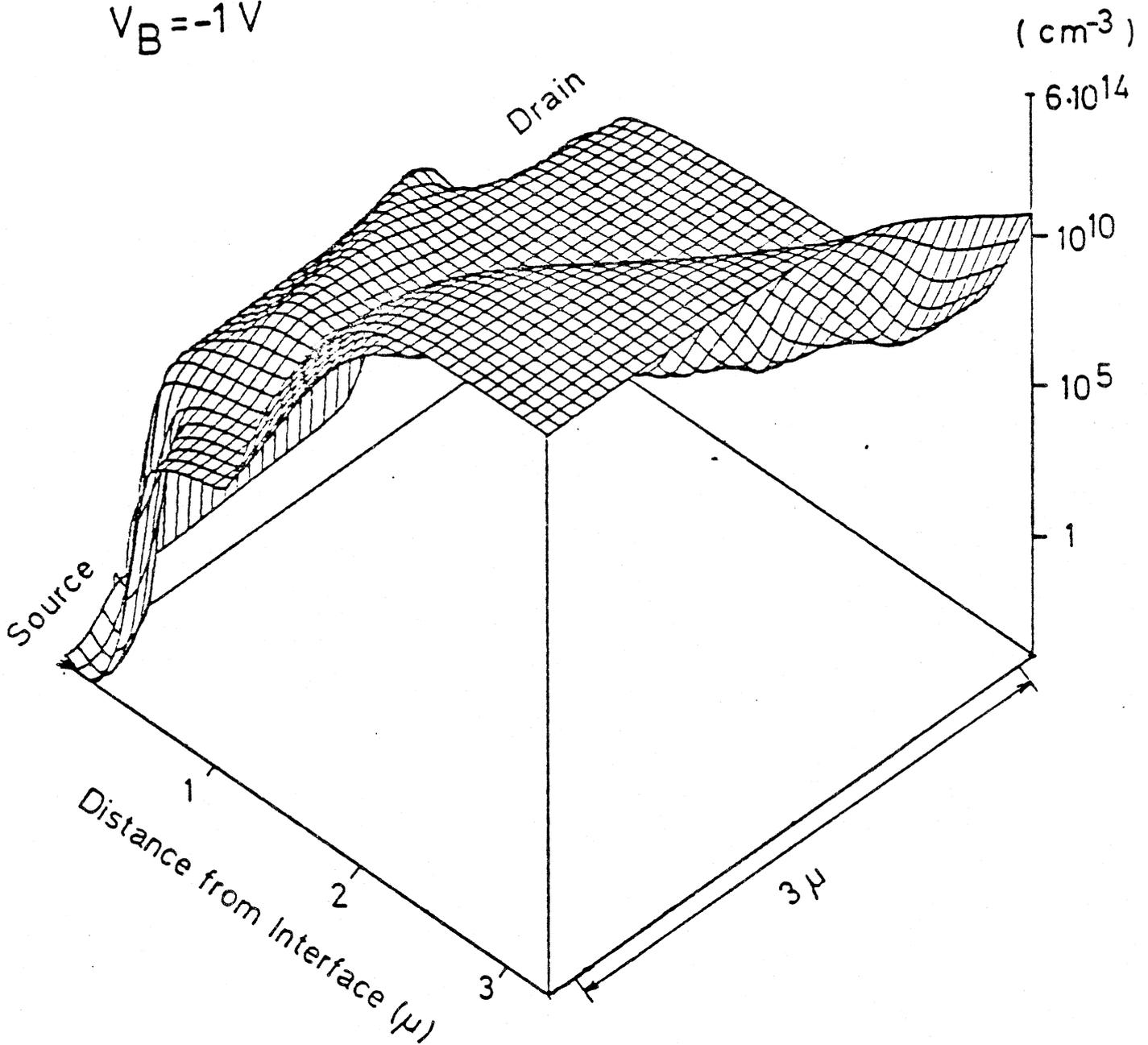
AVALANCHE GENERATION

$$V_G = 2V$$

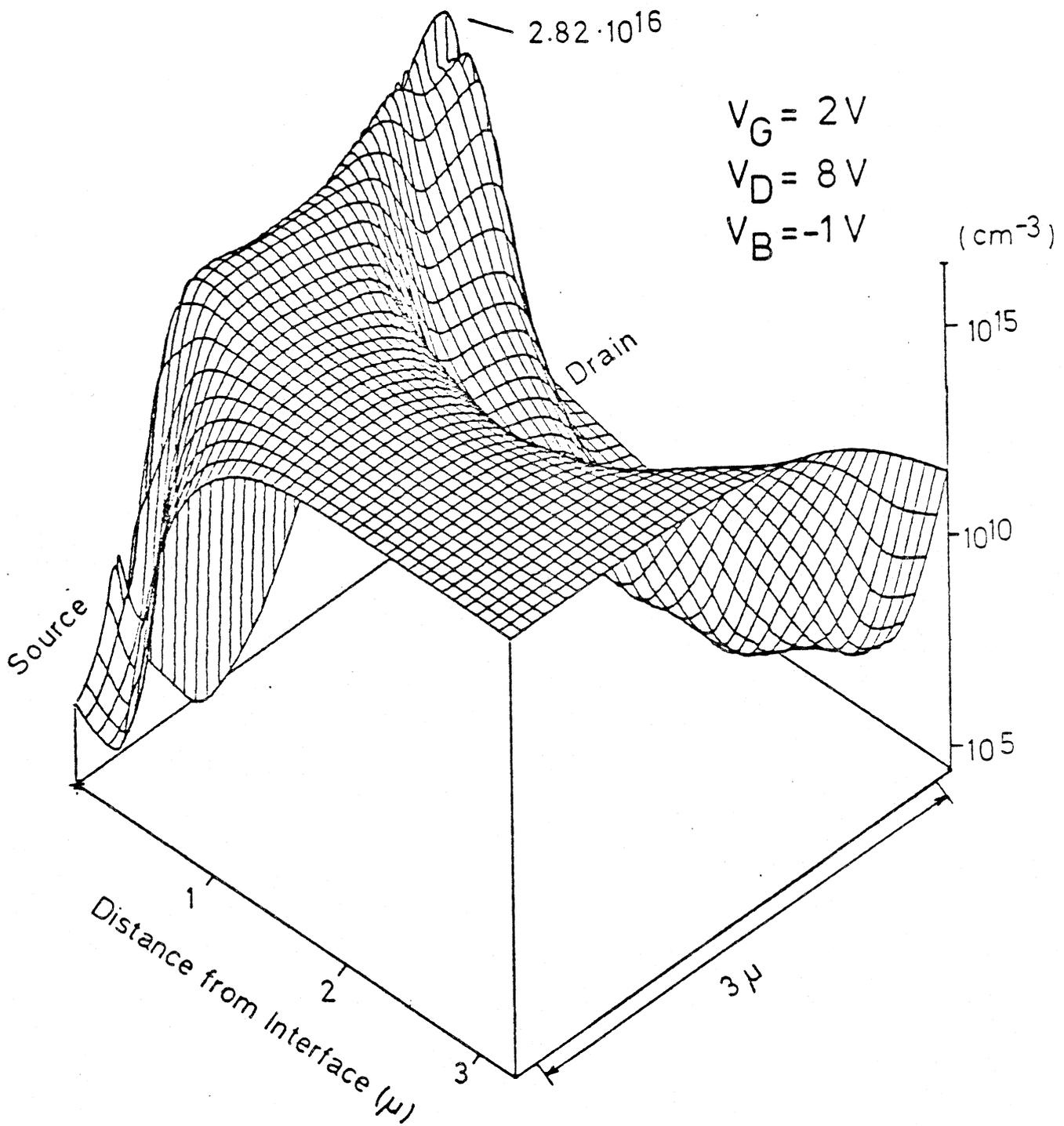
$$V_D = 8V$$

$$V_B = -1V$$

$$\alpha_{n,p} = 0$$



HOLE DISTRIBUTION



# HOLE DISTRIBUTION