Closed-Form Electrostatic Field Analysis for Metallic Comb-like Structures Containing Single and Interconnected Floating Strips of Arbitrary Topological Complexity
Part I: One-Dimensional Representation

Ali R. Baghai-Wadji
Siegfried Selberherr
Franz J. Seifert

Institut für Allgemeine Elektrotechnik und Elektronik
Abteilung für Angewandte Elektronik
Gußhausstr. 27-29, A-1040 Wien
AUSTRIA

I. Introduction

It is well known that the frequency response and radiation characteristic of surface acoustic wave (SAW) interdigital transducers, consisting of a set of parallel thin metallic strips (fingers) deposited on a piezoelectric substrate, can satisfactorily be described, if the charge distribution on the fingers is determined, [1]. Further, in linear weak piezoelectric approximation, the Fourier transform of the electrostatic charge distribution can be regarded as the driving function for SAWs, which is the most relevant function for the design of SAW filters, [2].
In the past several years considerable research effort has been devoted to the development of methods for the field analysis applicable to SAW transducer structures, [3], [4], [5]. Recently new SAW devices have been introduced, which contain single and interconnected floating fingers (FF) to achieve a desired frequency response, [6]. However, since the potentials of the FF's are a priori unknown, and the boundary conditions generally have a complicated nature, the inclusion of FF's in the analysis of SAW filters has not been possible so far.
Using the Green's function formulation, the spectral domain representation and the moment method, we have developed a non-iterative, semi-numerical method of analysis with closed-form formulae. The involved integral equation is replaced by a matrix equation. The elements of the resulting matrix, a modified inverse capacitance matrix, are calculated in closed form. The electrode interaction and the end-effects are fully taken into account.

II. Theory

Consider a semi-infinite anisotropic dielectric. On the surface of the substrate a finite number of infinitely thin metallic strips (fingers) with ideal conductivity may be deposited. The fingers are infinitely long and parallel to the x-axis ($\frac{\partial}{\partial x} \equiv 0$). The width of the fingers and the spacing between any two fingers may be arbitrary. There are no restrictions imposed on the finger potentials, they may be real- or complex-valued, and not all the fingers have to be driven electrically, that is may float. The problem is to find an efficient and rigorous solution for the boundary value problem sketched in Fig.1.

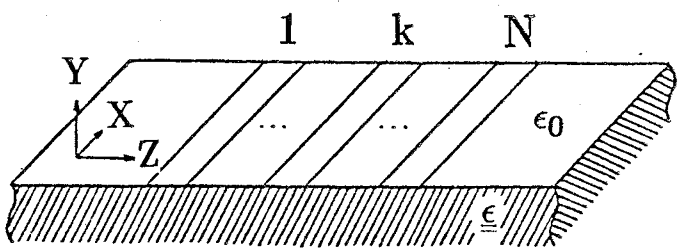

Fig.1:One dimensional representation of a SAW Transducer

By substitution one can show that solutions for the electric potential $\phi(y,z)$ of the form

$$\phi(y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{\phi}(k_z)\cdot e^{-jk_z z}\cdot e^{\psi(y,k_z)\cdot y}dk_z + C \qquad (II.1)$$

satisfy the Laplace's equation in the free space and within the substrate. Therein,

$$\psi(y,k_z) = -|k_z|\cdot H(y) + (j\frac{\epsilon_{23}}{\epsilon_{22}}k_z + \frac{\epsilon_p}{\epsilon_{22}}|k_z|)\cdot H(-y), \qquad (II.2)$$

H(y) is the Heaviside's stepfunction and $\epsilon_p = \sqrt{\epsilon_{22}\epsilon_{33} - \epsilon_{23}{}^2}$. Once we have found an expression for $\overline{\phi}(k_z)$, then the potential distribution $\phi(y,z)$ in the whole (R_y, R_z)–space is uniquely determined. Furthermore, by means of $\vec{E} = -\vec{\nabla}\phi$, $\vec{D} = \epsilon_0\cdot\vec{E}$ in the free space and $\vec{D} = (\underline{\epsilon})\cdot\vec{E}$ within the substrate the vectors $\vec{E}(y,z)$ and $\vec{D}(y,z)$ can be calculated from $\phi(y,z)$ directly. (II.1) with $y=0$ gives

$$\Phi(z) = \phi(0,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{\phi}(k_z)e^{-jk_z z}dk_z + C \qquad (II.3)$$

Eq.(II.3) shows that $\overline{\phi}(k_z)$ is the Fourier transform of the potential distribution on the surface, $\Phi(z)$. The boundary condition at the surface $(y=0)$ is

$$D_y(0^+,z) - D_y(0^-,z) = \rho(z) \qquad (II.4)$$

Where D_y is the y-component of the electric displacement.

A general expression for the charge distribution on the fingers is the corresponding Fourier integral.

$$\rho(z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{\rho}(k_z) e^{-jk_z z} dk_z \qquad (II.5)$$

(II.4) with regard to (II.1) and (II.5) gives

$$\overline{\phi}(k_z) = \overline{G}_e(k_z) \cdot \overline{\rho}(k_z) \qquad (II.6)$$

where the bar denotes the Fourier transform and

$$\overline{G}_e(k_z) = \frac{\alpha_e}{|k_z|}, \qquad \alpha_e = \frac{1}{\epsilon_0 \cdot (1 + \epsilon_p)} \qquad (II.7)$$

Insertion of (II.6) in (II.1) results in (II.8)

$$\Phi(z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{G}_e(k_z) \cdot \overline{\rho}(k_z) \cdot e^{-jk_z z} dk_z + C. \qquad (II.8)$$

Assuming that the inverse Fourier transform of $\overline{G}_e(k_z)$ can be formulated as follows in (II.10), and denoting it by $G_e(z)$, (II.8) can be written as

$$\Phi(z) = \int_{-\infty}^{+\infty} G_e(z\prime - z)\rho(z\prime)dz\prime + C. \qquad (II.9)$$

The inverse Fourier transform of $\overline{G}_e(k_z)$. is defined by

$$G_e(z) = \frac{1}{2\pi} \oint_{-\infty}^{+\infty} \overline{G}_e(k_z) \cdot e^{-jk_z z} dk_z \qquad (II.10)$$

where the integral symbol in (II.10) means that the integration has to be interpreted in Cauchy's sense. Using the definition of the Cauchy's integral, we have

$$G_e(z) = -\frac{\alpha_e}{\pi} \cdot \gamma - \frac{\alpha_e}{\pi} \cdot \lim_{\epsilon \to 0} ln\epsilon - \frac{\alpha_e}{\pi} \cdot ln|z|, \qquad (II.11)$$

where γ is the Euler Constant. Notice that although $G_e(z)$ not even exists in Cauchy's sense, the expression in (II.11) allows further algebraical manipulations, as follows: Insertion of (II.11) in (II.9) gives

$$\Phi(z) = \int_{-\infty}^{+\infty} \left(-\frac{\alpha_e}{\pi} \cdot \gamma - \frac{\alpha_e}{\pi} \cdot \lim_{\epsilon \to 0} \cdot ln\epsilon - \frac{\alpha_e}{\pi} \cdot ln|z\prime - z| \right) \rho(z\prime)dz\prime + C \qquad (II.12)$$

The first and the second parts in $(II.12)$ are independent of $z\prime$ and because of the charge neutrality condition ($\int_{-\infty}^{+\infty} \rho(z\prime)dz\prime \equiv 0$) we obtain

$$\Phi(z) = -\frac{\alpha_e}{\pi} \cdot \int_{-\infty}^{+\infty} ln|z\prime - z|\rho(z\prime)dz\prime + C \qquad (II.13)$$

III. Discretization of the Fingers

To reduce the number of the discretization points of the fingers (saving computer memory requirements and calculation times) we have chosen the following non-equidistant discretization scheme. Let an arbitrary finger in a given transducer have the start and end coordinates z_s and z_e. The width of the finger, may be denoted by $l = z_e - z_s$, Fig.2.

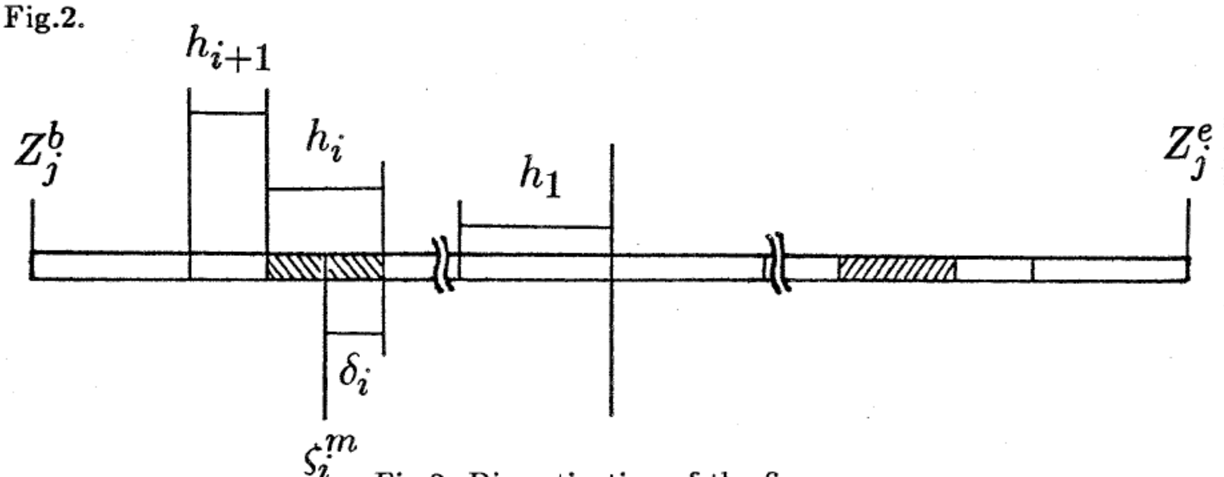

Fig.2: Discretization of the fingers

On the width of the substrips of the fingers the conditions $h_{i+1} = \alpha \cdot h_i$, $i = 1..n-1$, may be imposed, where $0 < \alpha < 1$. For given z_s, z_e, α and M, h_i (i= 1..n) can be calculated from (III.1).

$$h_1 + ... + h_n = \frac{l}{2}; \quad h_{i+1} = \alpha \cdot h_i; \quad i = 1 \,...\, n-1. \qquad (III.1)$$

Then with regard to the Fig.1 one can easily evaluate the ς_i^m and δ_i.

IV. Approximation of the charge density on the fingers

In this section we give a formula for the stepfunction approximation of the charge density. Other approximations, which are frequently used in the moment method technique are triangle and pulsfunction approximations. Formally the stepfunction approximation for the j^{th} substrip can be written as

$$\rho_j(z) = \sigma_j \cdot [H(z - z_j^b) - H(z - z_j^e)] \qquad (IV.1)$$

Now let us assume that the transducer under consideration consists of N substrips. Using (IV.1), the stepfunction approximation for the charge density is

$$\rho(z) = \rho_0 \sum_{j=1}^{N} \sigma_j \cdot [H(z - z_j^b) - H(z - z_j^e)] \qquad (IV.2)$$

V. Approximation of the potential distribution on the surface

Insertion of (IV.2) in (II.13), setting $\frac{\alpha_e}{\pi}\rho_0 = 1$ and interchanging the order of the summation, we have

$$\Phi(z) = \sum_{j=1}^{N} \sigma_j \cdot \int_{-\infty}^{+\infty} \ln|z\prime - nz| \cdot [H(z\prime - z_j^b) - H(z\prime - z_j^e)]dz\prime + C \qquad (V.2)$$

With regard to the definition of Heaviside's function, and the explicit formulation of the charge neutrality condition, evaluating the integrals we obtain

$$\Phi(z) = \sum_{j=1}^{N} \sigma_j \cdot [-(z_j^e - z)ln|z_j^e - z| + -(z_j^b - z)ln|z_j^b - z|] + C \qquad (V.3)$$

VI. Point-Matching and a Modified Inverse Capacitance Matrix

In the preceding section we have found an approximation for $\Phi(z)$, which contains $N+1$ unknowns. The unknowns are the N constant charge values on the substrips $(\sigma_j, j = 1..N)$ and the parameter C. The physical meaning of C will be given below. One possible way to determine σ_j and C is the so-called point-matching or collocation method, which will be discuussed in this section. A systematic derivation of the formulae demands firstly to assume, that all the fingers are driven electrically (there are no FF). In a later section the formalism will be slightly modified to include the class of SAW problems, which contain FF. Now let the potential of the i^{th} substrip be denoted by ϕ_i. Then the following is valid (Fig.2)

$$\Phi(z) = \phi_i, \quad z\epsilon[z_i^b, z_i^e] \qquad (VI.1-a)$$

$$\Phi(\xi_i^m) = \phi_i \qquad (VI.1-b)$$

Using the formula (V.2) for $\Phi(z)$ and (VI.1-b) we have

$$\phi_i = \sum_{j=1}^{N} \sigma_j \cdot (z_j^e - z_j^b) \cdot [-\frac{(z_j^e - \xi_i^m)}{z_j^e - z_j^b} \cdot ln|z_j^e - \xi_i^m| + \\ + \frac{(z_j^b - \xi_i^m)}{z_j^e - z_j^b} \cdot ln|z_j^b - \xi_i^m|] + C; \quad i = 1..N \qquad (VI.2)$$

With $\sigma_j \cdot (z_j^e - z_j^b) = q_j$, (VI.2) can be written

$$\phi_i = \sum_{j=1}^{N} q_j A^*(i,j) + C; \qquad i = 1 \ldots N \qquad (VI.3)$$

Where A^* is the inverse capacitance matrix. For a transducer with geometrical symmetric and electrical antisymmetric fingers, C is exactly zero. If the transducer considered relative to the reference transducer has a structural mismatch, C is not equal zero. Therefore C can be regarded as a system mismatch parameter. Equating $C = q_{N+1}$ and $A^*(i, N+1) = 1$ we can write

$$\phi_i = \sum_{j=1}^{N+1} q_j \cdot A^*(i,j); \qquad i = 1 \ldots N \tag{VI.4}$$

The explicit formulation of the charge neutrality condition $\sum_{j=1}^{N} q_j = 0$ with (VI.4) are N+1 equations for the N+1 unknowns q_j, which compactly is written as

$$\underline{\phi}^* = (\underline{\underline{A}}^*)\underline{q}^* \tag{VI.5}$$

VII. Method of Moments

The resulting formula (VI.2) section is quite general and can be applied to any transducer of arbitrary topological complexity and of unrestricted values of the finger potentials. However, with respect to a change of indices i and j ($i \to j, j \to i$), there is a lack of symmetry, which can be avoided as follows: The potential of the i^{th} substrip is now calculated by the mean value

$$\phi_i = \frac{1}{z_i^e - z_i^b} \cdot \int_{z_i^b}^{z_i^e} \Phi(z)dz. \tag{VII.1}$$

Insertion of $\Phi(z)$, (V.2), in (VII.1) and performing the integration we obtain (VII.2), where L is a normalization factor.

$$\begin{aligned} A(i,j) = & \frac{1}{2} \frac{1}{(\frac{z_i^e - z_i^b}{L}) \cdot (\frac{z_j^e - z_j^b}{L})} \cdot \\ & \cdot [\ + (\frac{z_j^e - z_i^e}{L})^2 \cdot ln|\frac{z_j^e - z_i^e}{L}| - \\ & - (\frac{z_j^e - z_i^b}{L})^2 \cdot ln|\frac{z_j^e - z_i^b}{L}| - \\ & - (\frac{z_j^b - z_i^e}{L})^2 \cdot ln|\frac{z_j^b - z_i^e}{L}| + \\ & + (\frac{z_j^b - z_i^b}{L})^2 \cdot ln|\frac{z_j^b - z_i^b}{L}|\ ]. \end{aligned} \tag{VII.2}$$

or (VI.5) in a symmetrical form:

$$\underline{\phi}^{**} = (\underline{\underline{A}}^{**})\underline{q}^{**} \tag{VII.3}$$

VIII. Floating Fingers

In some cases of greatest practical interest, [6], to control more efficiently the frequency response and the radiation characteristic of SAW filters, a number of the fingers are disconnected from the bus bars. These FF's can be single, SFF's or interconnected in groups, ICFF's. The aim of this section is to include the SFF's and ICFF's in the analysis. The system of equations, which is derived in the two preceding sections have to be modified as follows: The apriori unknown potentials of the single and/or interconnected floating fingers can be regarded as additional components of the unknown charge vector. Doing that, the number of the columns increases according to the number of floating fingers, so that we need further equations to make the system of equations invertable. The additional equations are the explicit formulation of the charge neutrality conditions for SFF and ICFF, which are included to the system of the equations as additional rows. The solution of the resulting system yields simultaneously the charge values on the N strips (SFF and ICFF included), the constant parameter C and the potentials of FF and ICFF.

Results

To demonstrate the generality of the described method, we have analyzed an hypothetical SAW interdigital transducer sketched in Fig.3. The transducer contains both SFF and ICFF.

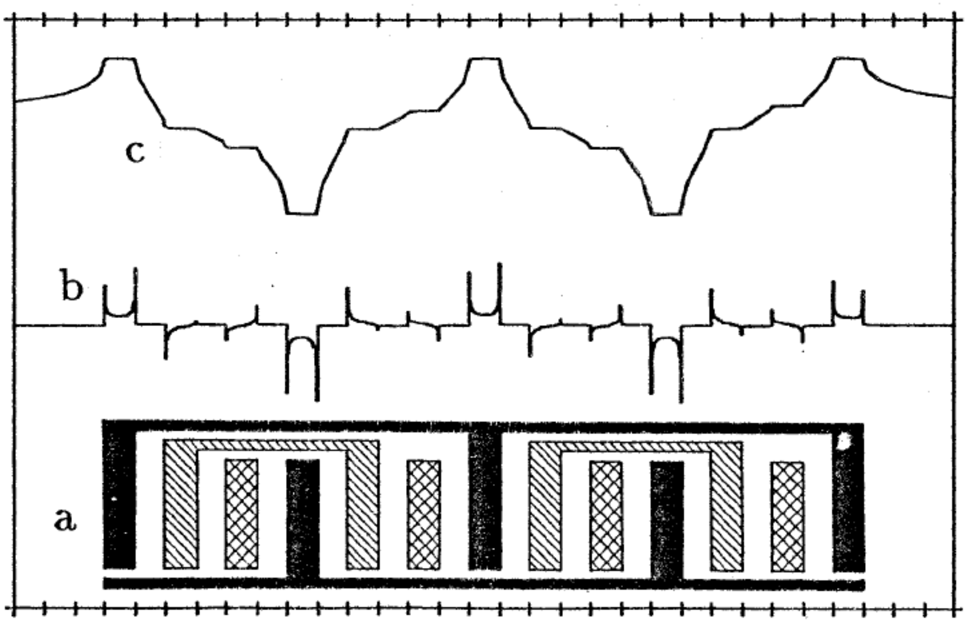

Fig.3 a: An hypothetical SAW interdigital transducer containing single and interconnected floating fingers, b: Charge distribution, c: Potential distribution

Conclusion

Based on Fourier transformation, the Green's function concept and Moment method we have shown that the electrostatic field problem for metallic comb-like structures containing single and interconnected floating strips can be solved with closed-form expressions.

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