CALCULATING COUPLING CAPACITANCES OF THREE–DIMENSIONAL INTERCONNECTIONS

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ABSTRACT

The capacitance, an integral quantity, can be calculated for some simple geometric structures analytically. The capacitance extraction in nonplanar VLSI structures leads to an electrostatic field calculation. For the design of line drivers, sensing amplifiers, data and bit lines of memories, the exact knowledge of all parasitic capacitances of a memory cell is essential. Therefore we have chosen the method of finite elements, since the capacitance extraction with the electrostatic energy which can be directly obtained from the variational integral is much more precise than a charge integration of the conductors.

VARIATIONAL FORMULATION

The calculation is based on the variational formulation $I \rightarrow min$ of Eq. (1), which is equivalent to the differential equation (2).

$$I_{min} = \int_G \varepsilon(x,y,z) \left(\left(\frac{\partial \psi}{\partial x} \right)^2 + \left(\frac{\partial \psi}{\partial y} \right)^2 + \left(\frac{\partial \psi}{\partial z} \right)^2 \right) dV \tag{1}$$

$$\text{div } \varepsilon(x,y,z) \text{ grad } \psi(x,y,z) = 0 \tag{2}$$

The functional I_{min} holds exactly twice the electrostatic field energy. For a charge–balanced, two conductor problem, the finite element formulation leads directly to the capacitance

$$C_{12} = \frac{2\, W_{el}}{U_{12}^2} \qquad W_{el} = \frac{1}{2}\, I_{min}. \tag{3}$$

With the finite element formulation a number of different materials can be handled easily. The boundary conditions and material interfaces are easy to implement. For the field calculation, the conductor boundaries represent Dirichlet boundary conditions, and the rest are homogenous Neumann conditions. For further development the method can be extended to regions with space charge to calculate nonlinear capacitances.

IMPLEMENTATION

With a three–dimensional transfinite interpolation [2], we obtain a boundary conforming grid which meshes various nonplanar interconnects. A discretization in hexahedron elements and additionally a hexahedron into tetrahedron element splitting were performed to avoid numerical integration of the element stiffness matrices.

Quadratic element shape functions for the ten–node tetrahedron elements allow a precise potential calculation for the energy determination. With these functions, each element holds a 10×10 elements stiffness matrix, which will be assembled in the global stiffness matrix. The next step is the handling of the boundary conditions. For the Dirichlet nodes we have chosen the method of dummy equations to preserve the structure of the stiffness matrix for

the node references. The global stiffness matrix is sparsely occupied. To achieve an efficient usage of computer memory, a compressed matrix format [1] is used. Only the nonzero row entries are stored. An additional index matrix holds the references to the column indices. To get an entry in the stiffness matrix, a binary search is used to find the column index in the index matrix. The example in Fig. 3 needs five searching steps on average. If a whole row or many entries of a row need to be manipulated, a temporary expansion of the row achieves a significant reduction of time consumption for assembling and solving the large linear system. These methods in combination with an iterative solver, make an efficient usage of computer resources possible with a relatively small effort.

EXAMPLES

There exist only a few simple really three–dimensional examples which can be calculated analytically to check the accuracy of the simulator. A reasonable alternative consists in checking two–and–half dimensional results which are calculated analytically or numerically.

A comparison of the example of Fig. 1 with our two dimensional Simulator VLSICAP [3], which is also based on finite elements for linear and nonlinear capacitances, shows a mismatch for the calculated coupling capacitance C_{mn} of 1 % (Fig. 2, 3) of this effectively two–dimensional problem.

The more realistic example in Fig. 4 shows the cross–section of two conductor wires above a grounded plane. Due to the symmetry only a quarter of the original configuration has to be considered. For this three–conductor example, we need three energy calculation runs with different conductor potentials, and have to solve a small linear equation system to extract the three partial capacitances C_{12}, C_{13} and C_{23}. A charge–balanced n conductor system has $n\ (n-1)/2$ partial capacitances. The capacitance in this example can only be obtained exactly with a three–dimensional tool. All simulations were carried out on a DECstation 5000.

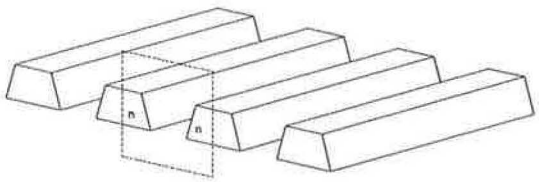

Figure 1: Parallel wires, rel. perm. 3.9

CONCLUSION

A first implementation of a three–dimensional tool for the calculation of linear capacitances in VLSI structures based on the finite element method has been performed.

The disadvantage of the numerical very stable method lies in the necessity of performing spatial discretization, which is a very complicated task, particularly in three dimensions.

REFERENCES

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[3] TRAAR, K. P.
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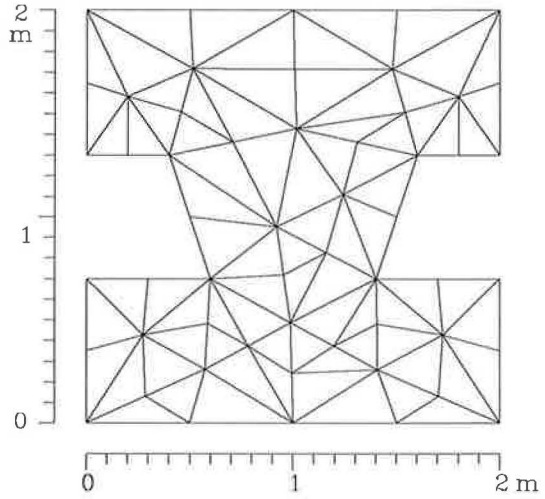

Figure 2: 2D–mesh $C_{mn} = 52.30pF/m$,
78 elements, 159 nodes

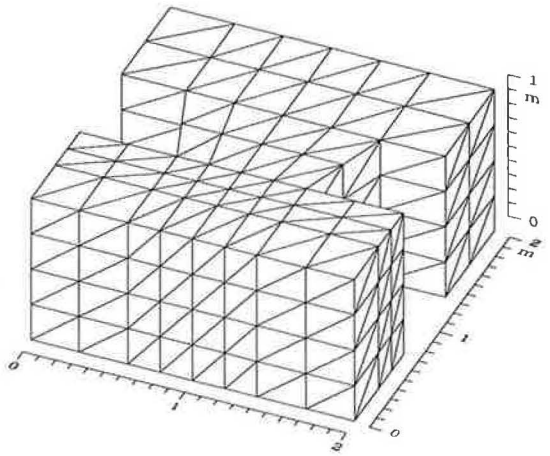

Figure 3: 3D–mesh $C_{mn} = 52.19pF$,
1056 elements, 1899 nodes,
76 sec. sim. time

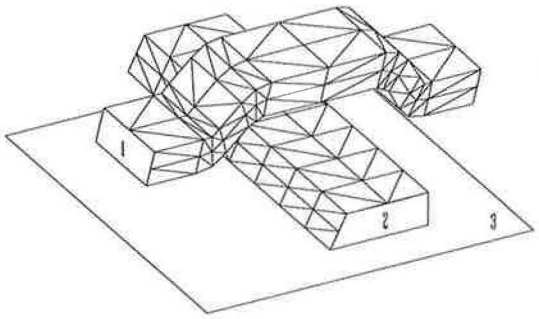

Figure 4: Crossing–lines,
rel. perm. 3.9

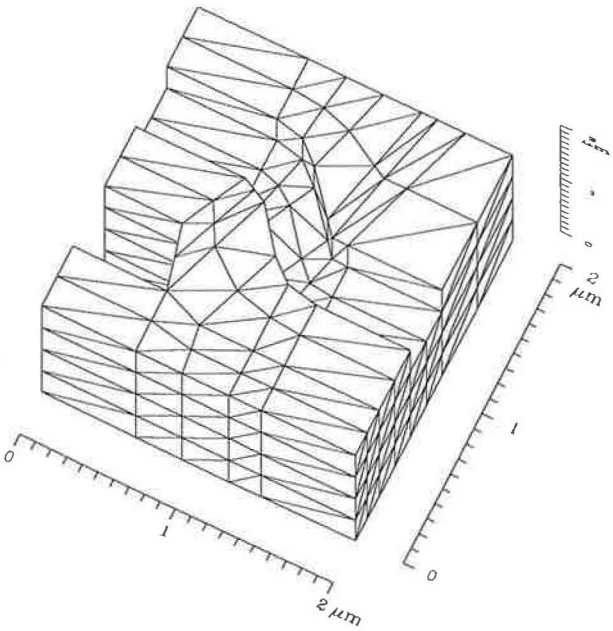

Figure 5: 3D–crossing–lines mesh,
$C_{12} = 4 \cdot 93.10fF$, $C_{13} = 4 \cdot 45.93fF$,
$C_{23} = 4 \cdot 140.0fF$. 1164 elements,
2079 nodes, 280 sec. sim. time