Calculation of the Radiation from the Slot of a Slim Enclosure with a Cavity Resonator Model

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***Abstract*—A closed form expression for the cavity field within a slim, cubical, metallic enclosure with three closed edges and one open edge is presented. The radiated far fields from the open slot are described by analytical formulations for the electric field and the radiated power. The formulations allow to efficiently analyze resonance frequencies, positions of maximum field density, and placement criteria for critical electronic components within the enclosure.**

I. Introduction

Cavity field models are established for the full wave description of power plane fields [1], [2] and the far field radiation calculation of rectangular power planes [3], [4].Such analytical formulations have a lot of advantages for practical applications:

- Fields can be calculated much faster than by numerical field simulation
- Resonance frequencies can be calculated from closed form expressions
- Positions of maximum field values can be found by discussion of the analytical expressions for a wide frequency range
- The influence of the position of electronic components can be investigated

Slim, metallic enclosures (Fig. 1) are required for many applications, such as (CD/DVD) drives, slots for 19'' racks, shielding covers for displays or other sensitive devices, or automotive control devices.

When the distance of the cover to the bottom of such devices is small compared to the wavelength, a cavity field formulation is applicable in order to describe the fields, e.g. [1] - [5].

The cavity field expressions in [1] [2] have been derived for rectangular planes with four open edges, assuming perfect magnetic conducting boundary conditions at the edges. An enclosure with metallic walls can be calculated with these formulations by introduction of a sufficient amount of low impedance components to ports at the edges. However, the different boundaries result in changes regarding resonance frequencies and maximum field position, which cannot be investigated by simple discussions of the analytical expression, because the boundary conditions are not satisfied by the basis functions of the analytical formulation. Therefore the last three advantages vanish.

In Section II we present a cavity field formulation, using perfectly electric conducting boundary conditions (PEC) for the closed edges and a perfect magnetic conducting boundary condition (PEM) for the open slot (Fig. 1).

We use this formulation for the discussion of maximum field positions, the resonance frequencies, and placement rules for critical electronic devices.

Closed form, approximate expressions for the electric field and the pointing vector in the far field are presented in Section III. We used the equivalent source method to obtain these expressions, similarly as shown for planes with four open boundaries by [3]. The closed metallic walls of the enclosure have influence on the radiated field distribution. However, comparisons with HFSS® (Ansoft®) simulations have shown that the contribution of the side walls to the radiated power can be neglected. Therefore, we can obtain the far field formulation only from the slot fields.

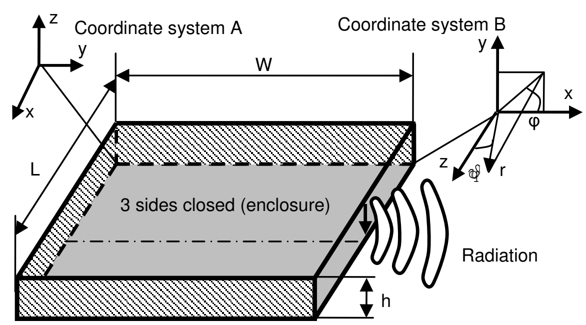

Fig. 1 A cubical metallic enclosure with three closed edges and one open slot. The cavity field inside the enclosure causes radiated emission from the open slot. Coordinate system A is used for the derivation of the cavity field expression in Section II and system B for the far field expressions in Section III.

II. Analytical Cavity Field Formulation For A Slim Enclosure with a slot on one side

For the derivation of the cavity field formulations within the enclosure depicted in Fig. 1 a vectorial Helmholz equation can be obtained from the general Maxwell equations for linear, isotropic, and homogenous field domains:

(1) $$\vec{\nabla}^2 \vec{E} + \omega^2 \mu \cdot \varepsilon \cdot \vec{E} = j \cdot \omega \cdot \mu \cdot \vec{J}_s$$

The upper and lower planes of the enclosure have highly electric conductivity. Together with the PEM boundary at the slot of the enclosure it is obvious that the field components *Ex* and *Ey* vanish, as long as the distance of the cover to the bottom plane is electrically short.

Therefore (1) can be reduced to a scalar equation:

(2) $$\vec{\nabla}^2 E_z + \omega^2 \cdot \mu \cdot \varepsilon \cdot E_z = j \cdot \omega \cdot \mu \cdot J_z$$

E_z *z component of the electric field (between the planes)*
J_z *z component of the current density*

Using the separation method, the solution of (2) is:

(3) $$E_z = X(x) \cdot Y(y)$$

With

(4) $$X(x) = A_m \cdot \sin(k_m \cdot x) + B_m \cdot \cos(k_m \cdot x)$$
(5) $$Y(y) = C_n \cdot \sin(k_n \cdot y) + D_n \cdot \cos(k_n \cdot y)$$

The boundary conditions are:

$x = 0$ & $x = L$ *PEC walls* $E_z = 0 \Rightarrow X(x) = 0$
$\Rightarrow B_m = 0$ & $k_m \cdot L = m \cdot \pi$

$y = 0$ *PEC walls* $E_z = 0 \Rightarrow Y(y) = 0$
$\Rightarrow D_n = 0$

$y = W$ *PEM walls* $\frac{\partial E_z}{\partial y} = 0 \Rightarrow \frac{\partial Y(y)}{\partial y} = 0$
$\Rightarrow k_n \cdot W = \frac{(2 \cdot n + 1)}{2} \cdot \pi$

PEM … perfect electric conducting boundary
PEM … perfect magnetic conducting boundary

This leads to the general solution:

(6) $$E_z = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left(L_{mn} \cdot \sin(k_m \cdot x) \cdot \sin(k_n \cdot y) \right)$$

With an excitation current at position xj, yj: Jz, Lmn can be calculated in the same way as for the derivation of the formulation in [1], [2], which leads finally to:

(7) $$Z_{ij} = \frac{j \cdot \omega \cdot \mu_0 \cdot h}{L \cdot W} \cdot \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left\{ \frac{4}{k_{x,m}^2 + k_{y,m}^2 - k^2} \sin(k_{x,m} \cdot x_i) \cdot \sin(k_{y,n} \cdot y_i) \cdot \sin(k_{x,m} \cdot x_j) \cdot \sin(k_{y,n} \cdot y_j) \right\}$$

(8) x_i, y_i, x_j, y_j *port positions of port i and j*

(9) $$k_{x,m} = \frac{m \cdot \pi}{L}, \quad k_{y,n} = \frac{(2 \cdot n + 1) \cdot \pi}{2 \cdot W}, \quad k = \frac{\omega}{c_l}$$

The voltage between the upper and the lower plane at position (xi, yi) is given by:

(10) $$U_i(x_i, y_i) = \sum_{j=1}^{n\_port} \left(Z_{ij}(x_i, y_i, x_j, y_j) \cdot I_j(x_j, y_j) \right)$$

$I_j(x_j, y_j)$ *current on port j*
n_port *number of ports with non zero current*

The electric field is given by:

(11) $$\vec{E}_i(x_i, y_i) = \frac{U_i(x_i, y_i)}{h} \cdot \vec{e}_z$$

The resonance frequencies of the enclosure are given by:

(12) $$f_r = \frac{c_0}{2 \cdot \pi} \cdot \sqrt{\left(\frac{m \cdot \pi}{L} \right)^2 + \left(\frac{(2 \cdot n + 1) \cdot \pi}{2 \cdot W} \right)^2}$$

This shows that an enclosure with only one open slot has different resonance frequencies as two planes with four open boundaries.

Resonance frequencies of the modes with m=0 do not exist, because the nominator of (7) vanishes at the same frequency as the denominator (sin() function). For example, an enclosure with *L=160mm* and *W=120mm* has the following resonance frequencies Table 1.

TABLE I
Resonance Frequencies of an Enclosure with One Open Edge

Mode		Resonance frequency	Exist
m	**n**	**MHz**	**yes/no**
0	0	625	no
1	0	1127	yes
0	1	1875	no
1	1	2096	yes

Table 2 contains the resonance frequencies of two planes with *L=160mm,W=120mm* and four open edges, which was calculated following [1]:

$$(13)\quad f_r = \frac{c_0}{2\cdot\pi}\cdot\sqrt{\left(\frac{m\cdot\pi}{L}\right)^2+\left(\frac{n\cdot\pi}{W}\right)^2}$$

TABLE II
RESONANCE FREQUENCIES OF TWO PLANES WITH FOUR OPEN EDGES

Mode		Resonance frequency	Exist
m	**n**	**MHz**	**yes/no**
0	0	-	no
1	0	938	yes
0	1	1250	yes
1	1	1563	yes

Table I and Table II show that the resonance frequencies of the cavity field depend significantly on the boundary conditions. Equation (12) offers a quick opportunity to estimate the resonance frequencies of a slim enclosure.

The sin() functions in expression (7) show directly that the maximum values of the field will be at the slot and also that the placement of a source near to the enclosure walls will reduce the emission of this source.

In the next section the radiated power dependence on the source position will be presented.

III. FAR FIELD EXPRESSIONS

We derived the analytical electric far field density formulation using the equivalent source method, similarly as it has been done in [3] for planes with open boundaries. In case of an enclosure the metallic walls have influence on the field distribution. For the exact consideration of this influence a boundary value problem for the exterior of the enclosure has to be solved by a field simulation program. To obtain an efficient design formulation, we neglected the influence of the metallic walls and obtained the far field formulations only from the slot fields. We compared the radiated power calculated with this formulation with three-dimensional full wave calculations, using HFSS®, which showed that the contribution of the enclosure walls to the radiated power can be neglected. Therefore the presented far field expressions can be used for the efficient calculation of the radiated power. The exact spherical field distribution cannot be calculated with these formulations, because the directivity, initiated by the enclosure walls is not taken into account. However, an approximate field distribution can be calculated, especially for the first resonance modes, where the directivity is low. Simulations with HFSS® showed low directivity especially up to the first resonance frequency of the enclosure. Therefore the analytical expressions have a considerable accuracy up to the first resonance. As the expressions depend only on q, significant radiated field density is not only in front of the slot. It can be measured in the whole x-y plane of coordinate system B in Fig. 1.

We used magnetic current sources on the slot to obtain the approximate field formulations (coordinate system B, Fig. 1):

$$(14)\quad \vec{E} \approx \frac{k\cdot\omega\cdot\mu_0\cdot h}{\pi\cdot W\cdot L}\cdot\frac{e^{-j\cdot k\cdot r}}{r}\cdot I_S\cdot\sin(\vartheta)\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{\sin(k_m\cdot z_S)\cdot\sin(k_n\cdot(x_S+W))}{k_m^2+k_n^2-k^2}\cdot(-1)^n\cdot\frac{k_m}{k_m^2-k^2\cdot\cos^2(\vartheta)}\cdot\left(1-(-1)^m\cdot e^{j\cdot k\cdot L\cdot\cos(\vartheta)}\right)\right]\right\}\cdot\vec{e}_\varphi$$

Using

$$(15)\quad \vec{\nabla}\times\vec{E} = -j\cdot\omega\cdot\mu\cdot\vec{H} \quad\Rightarrow\quad \vec{H} = \frac{1}{j\cdot\mu\cdot\omega}\cdot\frac{\partial_r(r\cdot E\varphi)}{r}\cdot\vec{e}_\vartheta$$

the magnetic far field density is:

$$(16)\quad \vec{H} \approx -\frac{k^2\cdot h}{\pi\cdot W\cdot L}\cdot\frac{e^{-j\cdot k\cdot r}}{r}\cdot I_S\cdot\sin(\vartheta)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{\sin(k_m\cdot z_S)\cdot\sin(k_n\cdot(x_S+W))}{k_m^2+k_n^2-k^2}\cdot(-1)^n\cdot\frac{k_m}{k_m^2-k^2\cdot\cos^2(\vartheta)}\cdot\left(1-(-1)^m\cdot e^{j\cdot k\cdot L\cdot\cos(\vartheta)}\right)\right]\Bigg\}\cdot\vec{e}_\vartheta$$

With:

$$\vec{S} = \vec{E}\times\vec{H}^*$$

The pointing vector in the far field region becomes:

$$(17)\quad \vec{S} \approx \frac{k^3\cdot h^2\cdot\mu_0\cdot\omega}{(\pi\cdot r\cdot W\cdot L)^2}\cdot I_S^2\cdot\sin^2(\vartheta)\left|\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\frac{\sin(k_m\cdot z_S)\cdot\sin(k_n\cdot(x_S+W))}{k_m^2+k_n^2-k^2}\cdot(-1)^n\cdot\frac{k_m}{k_m^2-k^2\cdot\cos^2(\vartheta)}\cdot\left(1-(-1)^m\cdot e^{j\cdot k\cdot L\cdot\cos(\vartheta)}\right)\right]\right|^2\cdot\vec{e}_r$$

We calculated the radiated emission for different source positions (Fig. 2, Fig. 3). To consider the radiation loss in the cavity field calculation we used a quality factor which we obtained by applying the method of [5].

(7) and (11) are used together with an effective wave number which introduces the radiation loss:

$$\mathrm{E}_z(x_i, y_i) = \frac{j \cdot \omega \cdot \mu_0}{L \cdot W} \cdot I_j \cdot \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\Bigg(\frac{K_{m,n}}{k_{x,m}^2 + k_{y,m}^2 - k_{eff}^{\;2}} \cdot \sin(k_{x,m} \cdot x_i) \cdot \sin(k_{y,n} \cdot y_i) \cdot \sin(k_{x,m} \cdot x_j) \cdot \sin(k_{y,n} \cdot y_j)\Bigg) \quad (18)$$

With:

$$k_{eff} = \frac{\omega}{c_l} \cdot \sqrt{1 - j \cdot \frac{1}{2 \cdot Q_r}} \quad (19)$$

Q_r ...*quality factor for radiation loss consideration*

As an example an enclosure with *L=160mm, W=120mm* and *h=7mm* has been used to calculate the radiation loss, depending on the source position. Fig. 2 and Fig. 3 show, that the emission from the slot can be significantly reduced by placing critical electronic components near to the metallic walls of the enclosure.

This is consistent to the fact that the sin() functions in (7) associated with the source position vanish at the metallic walls.

Any move of a device closer to the walls reduces the emission, however, the reduction is dominant close to walls, where the sin() variation has a maximum.

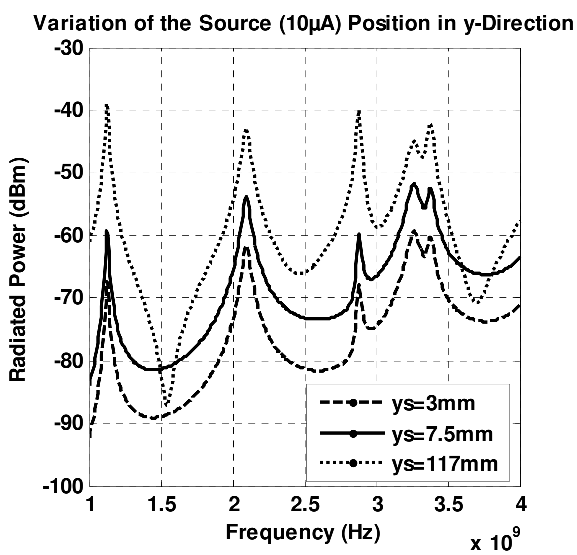

Fig. 2 Variation of the excitation source position (xs=80mm).The radiated power decreases, when the source is placed near the rear wall. Coordinates (xs, ys) corresponding to coordinate system A in Fig. 1.

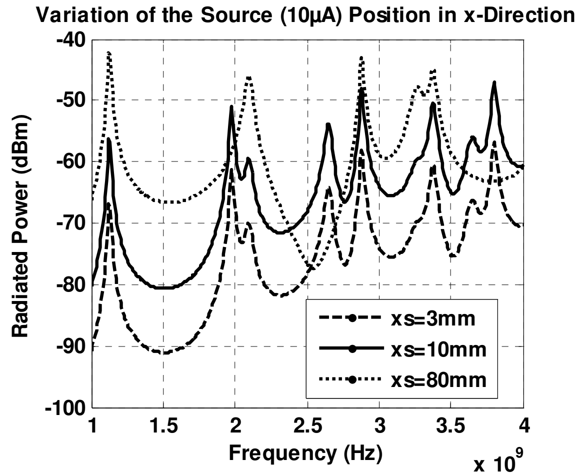

Fig. 3 Variation of the excitation source position (ys=60mm).The radiated power decreases, when the source is placed a near a side wall. Coordinates (xs, ys) corresponding to coordinate system A in Fig. 1.

IV. Conclusions

Closed form expressions have been presented for the cavity fields inside a cubical metallic enclosure with a slot on one side and for the far field densities.

These expressions have been used for the investigation of the resonance frequencies, the maximum field positions inside the enclosure, and position criteria for critical electronic devices.

The expressions can be used to get quickly information about the resonance frequencies, the influence of the source position, and the radiated power from the open slot.

Acknowledgment

This work was supported by the Medea+ Parachute program.

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