Green’s function asymptotic in two-layered periodic medium

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Abstract. Green’s function asymptotic properties of the wave field in a two-layered periodic medium is analytically examined. The solution is constructed by the stationary phase method. It is shown that some singularities occur in this approach. Namely, the Green’s function asymptotical form is proportional to the distance from a source as $R^{-5/6}$, instead of the expected $R^{-1}$ behavior. Thus, the power flux density in this directions decreases very slowly (as $R^{-5/3}$) and as a consequence there exists a set of ultra-propagation power channels. We focus on the description of the vicinity of a singular cone where the usual stationary phase method fails and determined a smooth transition from the standard asymptotic form to the new one. Finally, a detailed analysis of the obtained results is given.

Introduction

The Green’s function method applied to periodic structure has theoretical significance, but studying propagation and scattering of waves is one of the most effective methods to investigate systems with periodically varying in space properties [1,2]. This method is widely adopted in investigations of liquid crystals [3] and photonic band structures [4]. Here we study the field of a point source, i.e. the Green’s function in a two layered medium with a step-like spatial variation of wave numbers $(k_1, k_2)$ and one-dimensional periodicity for large distances from the source (Fig. 1). It is important to point out that the specified formulas do not depend on size and other structural parameters. It is shown that forbidden zones exist in such a media. The results are illustrated by numerical calculations.

1. Green’s function integral representation

We consider a scalar field and are not interested in polarization effects. The Green’s function $G(z, z_1, r)$ is the solution of the equation

$$\left(\Delta + k^2(z)\right)G(z, z_1, r) = -\delta(r - r_1),$$

which satisfies the limiting absorption principle. Taking advantage of the known integrated representation of the one-dimensional Green’s function [5,6] we receive the following asymptotic representation

$$G(z, z_1, r) \approx -\frac{1}{\sqrt{8\pi^3 r_\perp}} \int \frac{V(z_\perp)V^-(z_\perp)}{W(q)} \sqrt{q} dq \times \exp\left(iqr_\perp + p(q)|z - z_1| - \pi/4\right),$$

(1)

Fig. 1. Considered structure. The values $a_1, a_2, k_1, k_2$ are the layer widths and the wave numbers of the medium. $T$ is the periodicity.

2. Method of a stationary phase

The asymptotic form of the Green’s function in the far zone is to be considered, thus we calculate the integral (1) by the method of the stationary phase. In order to perform this it is necessary to find stationary points. The equation for definition of stationary points in polar coordinates (the origin is combined with a source $r_\perp = R \cos \varphi, |z - z_1| = R \sin \varphi$) looks like

$$p'(q) = -\text{ctg}\varphi.$$  \hspace{1cm} (2)

A typical quasi-momentum derivative curve $p'(q)$ is represented in Fig. 2.

Let the polar angle vary in the sector $\phi_* + \epsilon < \varphi < \pi/2$, where $\epsilon > 0$. Then the Green’s function asymptotic is defined by the unique stationary point $q = q_1$ and can be presented in the form

$$G(z, z_1, r, q_1) \approx \frac{V(z_\perp)V^-(z_\perp)}{2\pi W(q_1)} \sqrt{p'(q_1)r_\perp |z - z_1|} \times \exp\left(iqr_\perp + p(q_1)|z - z_1| + \pi/4 \text{sign}(p''(q_1)) - 1\right).$$ \hspace{1cm} (3)

3. Green’s function singularities

Received asymptotic (3) becomes inapplicable in a vicinity of $q = q_*, p'(q_*) = -\text{ctg}\varphi_*$. At this point the function $p''(q)$ is zero (special direction). In order to construct a smooth asymptotic form we rewrite (3) as

$$G(z, z_1, r_\perp, q_1) = A(z, z_1, r_\perp, q_1) \times \exp\left(iF(z, z_1, r_\perp, q_1)\right),$$

(4)

Fig. 2. The dispersion diagram $p(q)$ and quasi-momentum derivative curve $p'(q)$. The 1–2 dotted line shows the forbidden zone. It is visible, that the equation (2) has a unique root at $\text{ctg}\varphi = \text{ctg}\varphi_*$, two roots at $\text{ctg}\varphi = \text{ctg}\varphi_*$ and three roots in other cases. The angle $\phi_*$ is defined by the structure parameters.
Therefore, in the sector 0 function and power flux density are represented in Fig. 4. function for the periodic layered structure has a number of features. There are areas in which the wave field is described not only by a single wave, but by a sum of wave fields with the maximal number is defined by the properties of the structure and does not exceed number of extending normal waves for a wave guide consisting of two layers with periodic boundary conditions. Also, there exist allocated directions in which there is an occurrence or disappearance of an additional beam summand. The asymptotic of the wave field for these directions is described through the Airy function and in these directions the Green’s function and the power flux density decreases slowly. Such a structure can be used as an attractive optical material for controlling and manipulating the light flow. The obtained results offer additional features leading to new device concepts (e.g. microscale structure for the lightconfinement with radically different characteristics compared to conventional optical fiber [7]), when some technological aspects such as manufacturability and principal difficulties such as disorder are being under control.

References