# Numerical simulations of space charge waves in InP films and microwave frequency conversion under negative differential conductivity

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Numerical simulations of amplification and propagation of space charge waves in InP films is investigated theoretically. A microwave frequency conversion using the negative differential conductivity phenomenon is carried out when the harmonics of the input signal are generated. An increment in the amplification is observed in $n$-InP films at essentially higher frequencies $f < 70$ GHz, when compared with $n$-GaAs films $f < 44$ GHz. This work provides a way to achieve a frequency conversion and amplification of micrometer and millimeter waves. © *2011 American Institute of Physics*. [doi:10.1063/1.3555467]

The millimeter and submillimeter microwave ranges are very important for applications in communications, radar, meteorology, and spectroscopy. However, the structure of semiconductor devices (transistors, diodes, etc.), required for such a short wavelength, becomes very complex, which makes its fabrication difficult and expensive. One potential alternative to explore the use of such a part of the electromagnetic spectrum resides in the use of nonlinear wave interaction in active media. For example, the space charge waves in thin semiconductor films, possessing negative differential conductivity (InP, GaAs, GaN at 300 K and strained Si/SiGe heterostructures at 77 K), propagate at frequencies that are higher than the frequencies of acoustic and spin waves in solids. Space charge waves have been researched since a long time ago, which can be traced back to the 1950s.[1] The early experimental work on the amplification of space charge waves with a perturbation field started in the 1970s (Ref. [2]) and continues today.[3,4] The first monolithic device using space charge waves was a two-port amplifier developed in the beginning of 1970s in the United States. This device contained an $n$-GaAs film on a dielectric substrate, and a couple of source and drain ohmic contacts (OCs). A microwave signal applied to the input electrode modulates the electron density under this electrode. These modulations are drifted to the drain and amplified due to the negative resistance effect. The amplified signal is taken from the output electrode placed near the drain, see Fig. 1. Obviously, the output signal is maximal when all the waves reach the output electrodes with the same phase.[5]

Devices based on space charge waves use an attractive property of GaAs and InP. An electric field in excess of 15 kV/cm applied to an $n$-InP sample causes the differential electron mobility to become negative. To analyze wave phenomena in thin films of two-valley semiconductors,[6] a set of equations to describe the charge transport is commonly used. In this theory, with small initial perturbations, continuity, momentum and energy equations, and Poisson's equation are solved numerically. The solutions show that the modulations

of electron density travel along the beam in the form of waves called space charge waves. Although, this paper presents the numerical simulations of amplification and propagation of space charge waves in InP films, the experimental verification can be carried out in the same way like in GaAs films, already reported in Ref. [12].

The study of microwave frequency conversion under negative differential conductivity will be one of the most relevant topics in microelectronics and communications in the coming years, due to the potential it represents in terms of amplification of micrometer and millimeter waves. Therefore, we present two-dimensional (2D) numerical simulations of propagation and amplification of space charge waves
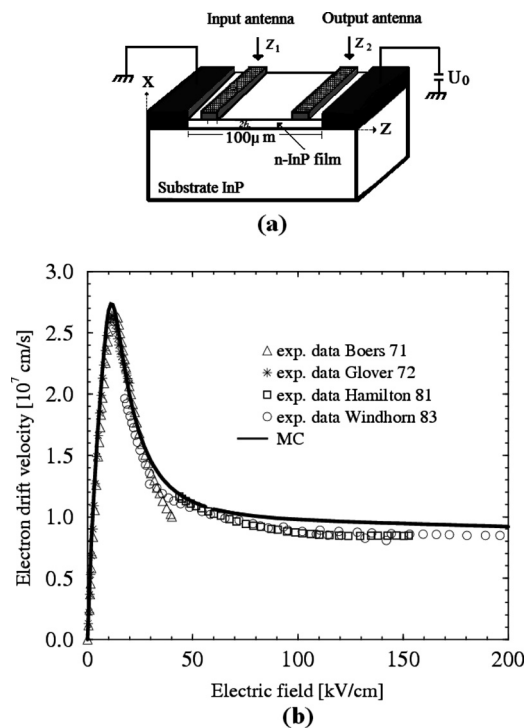


FIG. 1. (a) The structure of the $n$-InP traveling-wave amplifier fabricated with an epitaxial layer. (b) Electron drift velocity vs electric field; (-) own Monte Carlo simulation data vs. experimental data: (△) Boers (Ref. [8]), (*) Glover (Ref. [9]), (□) Hamilton (Ref. [10]), and (○) Windhorn (Ref. [11]).

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in InP films and microwave frequency conversion under negative differential conductivity phenomenon.

Consider $n$-InP film placed onto substrate without an acoustic contact. It is assumed that the electron gas is localized in the center of film. The thickness of the $n$-InP film is $2h \leq 1$ $\mu$m, see Fig. 1(a). The coordinate system is chosen as follows: X-axis is directed perpendicularly to the film, the electric field $E_0$ is applied along Z-axis, exciting and receiving antennas are parallel to Y-axis. 2D model of electron gas in the $n$-InP film is used. Thus, 2D electron concentration is presented only in the plane $x=0$. The space charge waves possessing phase velocity equal to drift velocity of the electrons $v_0=v(E_0)$, $E_0=U_0/L_z$, are considered, where $U_0$ is bias voltage, $L_z$ is the length of the film. Generally, a nonlocal dependence of drift velocity $v_d$ of electrons on the electric field takes place. In simulations, an approximation of 2D electron gas is used. The set of balance equations for concentration, drift velocity, and the averaged energy to describe the dynamics of space charge waves within the $n$-InP film takes a form, like in GaAs film[7]

$$\frac{d[m(w)\vec{v}_d]}{dt} = -q\left(\vec{E} - \frac{\vec{v}_d E_s}{v_s}\right); \quad \frac{dw}{dt} = -q(\vec{E}\vec{v}_d - E_s v_s);$$

$$\frac{\partial n}{\partial t} + div(n\vec{v}_d - D\nabla n) = 0; \quad D(w) = \frac{2}{3}\frac{\tau_p(w)}{m(w)}\left(w - \frac{1}{2}mv^2\right);$$

$$\vec{E} = \vec{e}_z E_0 - \nabla\varphi + \vec{e}_z \widetilde{E}_{ext}; \quad \Delta\varphi = \frac{q}{\varepsilon_0 \varepsilon}(n - n_0)\delta(x);$$

$$\widetilde{E}_{ext} = \sum_{j=1}^{2} E_{oj}\sin(\omega_j t)\exp\left[-\left(\frac{z-z_1}{z_0}\right)^2 - \left(\frac{y-y_1}{y_0}\right)^2\right] \quad (1)$$

where $v_d$ is drift velocity, $\varphi$ is the of potential, $n=n_0+\tilde{n}$, where $n_0$ is constant electron concentration, $\tilde{n}$ is the varying part, $w$ is the electron energy, $D$ is the diffusion coefficient, and $\varepsilon_0$ is the lattice dielectric permittivity of $n$-InP, $m(w)$ is averaged effective mass, $q$ is the electron charge, $\tau_{p,w}(w)$ are relaxation times, and $E_0$ is the bias electric field. It is assumed that a condition of occurring negative differential conductivity is realized. Because the signal frequencies are in microwave or millimeter wave range, it is possible to separate diffusion and drift motions. For the sake of simplicity, instead of relaxation times, the parameter $E_s$ is introduced,[7]

$$\frac{m(w)}{\tau_p(w)} = \frac{E_s}{v_s(E_s)}, \quad \frac{w-w_0}{\tau_w(w)} = qE_s v_s(E_s). \quad (2)$$

In such a representation, the mean energy and mean effective mass of electron are denoted by $w$ and $m(w)$, the equilibrium value of $w$ is $w_0$. A direct correspondence between local field dependence and nonlocal effects is well seen. Because a dependence $E_s=E_s(w)$ is unique, it is possible to express the parameters $w$ and $v_s$ through the value of $E_s$. The dependencies of drift velocity, averaged electron energy, and effective mass versus electric field in InP films were calculated by our Monte Carlo procedure, which are pretty similar as experimental results.[8–11] The drift velocity versus electric field is presented in Fig. 1(b).

The spatial increment of space charge waves is investigated by the dispersion equation, $D(\omega,k)=0$, the relation between angular frequency, $\omega=2\pi f$, and wave vectors, consider these like complex, $k=k'+ik''$, where $k'>0$. The
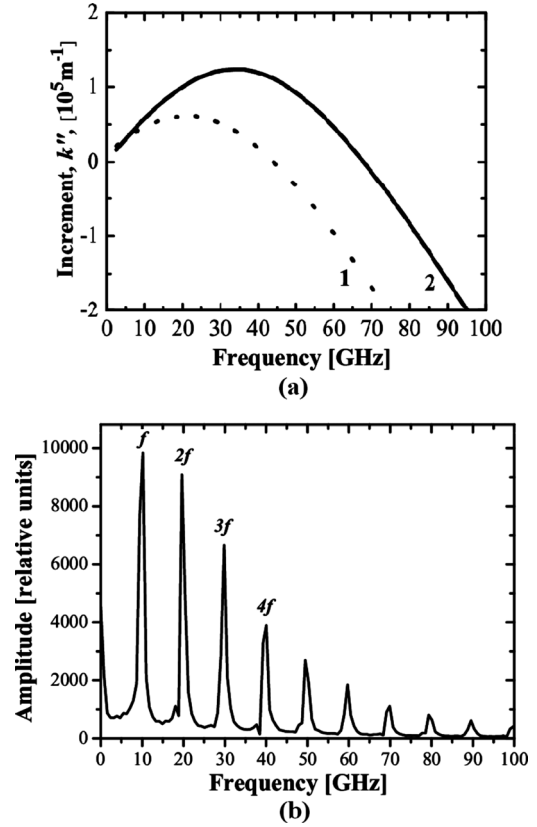


FIG. 2. (a) Spatial increments of instability $k''(f)$ of space charge waves. Curve 1 shows results for an $n$-GaAs film and curve 2 is for InP films with $E_0=20$ kV/cm, $n_0=5\times10^{14}$ cm$^{-2}$ and film thickness $2h=0.05$ $\mu$m using the balance equation model. (b) Spectral components of the electric field of space charge waves. The effective excitation of harmonics is presented. The input carrier frequency is $f=10$ GHz.

dispersion relation is obtained using balance equations model, set Eqs. (1), with some transformation we can obtain Eq. (3),

$$\frac{\partial\tilde{n}}{\partial t} + n_0\frac{\partial\tilde{v}}{\partial z} + v_0\frac{\partial\tilde{n}}{\partial z} - D\frac{\partial^2\tilde{n}}{\partial z^2} = 0, \quad (3)$$

but if we assume that $\tilde{n}$ obeys the law $\sim\exp(i\omega t - ikz)$, Eq. (3) gives the dispersion relation

$$[i(\omega - kv_0) + Dk^2]\tilde{n} - ikn_0\tilde{v} = 0. \quad (4)$$

In general, we consider the cases where $\omega=2\pi f$ is real and $k=k'+ik''$ has real and imaginary part. The case $k''>0$ corresponds to spatial increment (amplification), whereas the case $k''<0$ corresponds to the decrement (damping). In Fig. 2(a), the spatial increment of space charge waves in an $n$-GaAs film is shown in the curve 1, and the curve 2 shows the results for an $n$-InP film. It can be seen that an amplification of space charge waves in InP films occurs in a wide frequency range, and the maximal spatial increment is $k''=3\times10^5$ m$^{-1}$ at the frequency $f=35$ GHz. When compared with a case of the GaAs film, it is possible to observe an amplification of space charge waves in InP films at essentially higher frequencies $f>44$ GHz. To obtain an amplification of 25 dB, it is necessary to use a distance between the input and output antennas of about 0.09 mm.

When a small microwave electric signal $E_{ext} = E_m \cdot \sin(\omega t)\exp\{-[(z-z_1)/z_0]^2 - [(y-y_1)/y_0]^2\}$ is applied to the input antenna. Here $z_1$ and $y_1$ are the position of the input

antenna; $z_0$ and $y_0$ are its half-width. When this signal is applied, the excitation of space charge waves in 2D electron gas takes place. These waves are subject to amplification, due to negative differential conductivity.

The set of Eqs. (1) form a set of nonlinear coupled time dependent partial differential equations. These differential equations are discretized using a finite-difference scheme and are solved numerically. A transverse inhomogeneity of the structure in the plane of the film along Y axis is taken into account. The following parameters have been chosen: 2D electron concentration in the film is $n_0 = 5 \times 10^{14}$ cm$^{-2}$, the initial uniform drift velocity of electrons is $v_0 \approx 2 \times 10^7$ cm/s ($E_0 = 15 - 20$ kV/cm), the length of the film is $L_z = 0.05 - 0.1$ mm, the thickness of the film is $2h = 0.1 - 1$ $\mu$m. The typical output spectrum of the electromagnetic signal is given in Fig. 2(b). The input carrier frequency is $f = 10$ GHz. The amplitude of the input electric microwave signal is $E_m = 25$ V/cm. Although the growth rate decreases as the rf frequency increases, for our case an amplification of 25 dB is obtained. The duration of the input pulse is $2t_0 = 2.5$ ns.

The propagation and amplification of space charge waves in *n*-GaAs thin films with negative difference conductance have been studied in the past decade,[12,13] however *n*-InP films have not been addressed yet, and are subject of this work. We address the device presented in Fig. 1(a) by means of numerical simulations. An *n*-InP epitaxial film of thickness $0.1 - 1$ $\mu$m is put on an InP semi-insulating substrate. The 2D electron density in the film is chosen to be $n_0 = 5 \times 10^{14}$ cm$^{-2}$. On the film surface are the cathode and anode OCs, together with the input and output coupling elements (CEs). Designed as a Schottky-barrier strip contacts, the CEs connect the sample structure to microwave sources. A dc bias voltage (above the Gunn threshold, 20 kV/cm) was applied between the cathode and anode OCs, causing negative differential conductivity in the film. The CEs perform the conversion between electromagnetic waves and space charge waves, where the excitation of space charge waves in the 2D electron gas takes place. In the simulations an approximation of 2D electron gas is used.

One can see both the amplified signal at the first harmonic of the input signal and the harmonic generations of the input signal, which is generated due to the nonlinearity of space charge waves. The spatial distributions of the alternate component of the electric field $E_z$ and $E_y$ are shown in Fig. 3. One can see the maximum variations are in the output antenna. The length of the film is 0.1 mm. The transverse width of the film along Y axis is 1 mm. The duration of the input electric pulse is 2.5 ns. The spatial distributions are presented for the time moment 1.5 ns after the maximal value of the input signal. Direct numerical simulations have confirmed pointed below results on linear increments of space charge waves amplification. Also a possibility of nonlinear frequency doubling and mixing is demonstrated. To get the effective frequency doubling in the millimeter wave range, it is better to use the films with uniform doping.

A numerical simulation of 2D propagation and amplification of space charge waves in *n*-InP films is presented. A microwave frequency conversion using the negative differential conductivity phenomenon is carried out when the harmonics of the input signal are generated. A comparison of the calculated spatial increment of instability of space charge
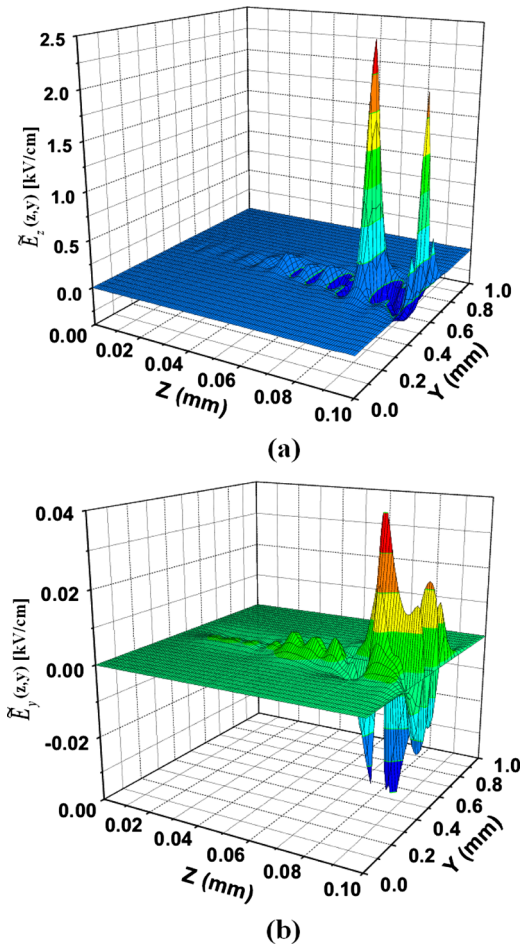


**(a)**



**(b)**

FIG. 3. (Color online) The spatial distributions of the alternative part of the electric field component $E_z$ (a) and $E_y$ (b) of space charge wave (a); The length of the film is 0.10 mm. The transverse width of the film along Y axis is 1 mm.

waves in *n*-GaAs and *n*-InP films is performed. An increment in the amplification is observed in InP films at essentially higher frequencies $f > 44$ GHz than in GaAs films. The maximum amplification (gain of 25 dB) is obtained at $f = 35$ GHz, using a distance between the input and output antennas of about 0.09 mm.

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