Short communication

Numerical simulation of amplification of space charge waves in n-InP films

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The non-linear interaction of space charge waves including the amplification in microwave and millimeter wave range in n-InP films, possessing the negative differential conductance phenomenon, is investigated theoretically. Both the amplified signal and the generation of harmonics of the input signal are demonstrated, which are due to non-linear effect of the negative differential resistance. It is possible to observe an amplification of the space charge waves in n-InP films of submicron thicknesses at essentially higher frequencies f < 70 GHz, when compared with n-GaAs films f < 44 GHz. The increment observed in the gain is due to the larger dynamic range in n-InP than in n-GaAs films.

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1. Introduction

The millimeter and sub-millimeter microwave ranges are very important for applications in communications, radar, meteorology and spectroscopy. However, the structure of semiconductor devices (transistors, diodes, etc.), required for such a short wavelength, becomes very complex, which makes its fabrication difficult and expensive. One potential alternative to explore the use of such a part of the electromagnetic spectrum resides in the use of non-linear wave interaction in active media. For example, the space charge waves in thin semiconductor films, possessing negative differential conductivity (InP, GaAs, GaN at 300 K and strained Si/SiGe heterostructures at 77 K), propagate at frequencies that are higher than the frequencies of acoustic and spin waves in solids. This means, for example, that an elastic wave resonator operating at a given frequency is typically 100,000 times smaller than an electromagnetic wave resonator at the same frequency. Thus attractively small elastic wave transmission components such as resonators, filters, and delay lines can be fabricated.

The scope of space charge waves’ applications is very large, because it can be useful to implement monolithic phase shifters, delay lines, and analog circuits for microwave signals. Space charge waves have been researched since a long time ago, which can be traced back to the 1950s [1]. The early experimental work on the amplification of space charge waves with a perturbation field started in the 1970s [2] and continues today [3,4]. The first monolithic device using space charge waves was a two-port amplifier developed in the beginning of 1970s in the United States. This device contained an n-GaAs film on a dielectric substrate, and a couple of source and drain ohmic contacts. A microwave signal applied to the input electrode modulates the electron density under this electrode. These modulations are drifted to the drain and amplified due to the negative resistance effect. The amplified signal is taken from the output electrode placed near the drain, see Fig. 1. Obviously, the output signal is maximal when all the waves reach the output electrodes with the same phase [5]. The majority of devices based on space charge waves are fabricated on GaAs films [1–5], although InP is recognized to have superior characteristics compared to GaAs for power generation in the millimeter wave range [17,18]. The threshold electric field is 3.2 kV/cm for gallium arsenide and 10 kV/cm for indium phosphide. The peak electron drift velocity is about 2.2 × 10^7 cm/s for gallium arsenide and 2.5 × 10^7 cm/s for indium phosphide. The maximum negative differential mobility is about −2400 cm^2/V·s for GaAs and −2000 cm^2/V·s for indium phosphide [19]. An electric field in excess of 10 kV/cm applied to an n-InP sample causes the differential electron mobility to become negative. To analyze wave phenomena in GaAs films semiconductors, a set of equations to describe the charge transport is commonly used [6]. In the case of InP, the same set of equations can be used, because the band structure of InP is very similar to the one of GaAs. In this theory, with small initial perturbations, continuity, momentum and energy equations, and Poisson’s equation are solved. The solutions show that the modulations of electron density travel along the beam in the form of waves called space charge waves; these results, for n-GaAs thin films, are in Ref. [20]. In this paper, the non-linear interaction of space charge waves including the amplification in
microwave and millimeter wave range in InP films, possessing the negative differential conductance phenomenon, is investigated theoretically; also the spatial increment of space charge waves in InP films is compared with the results in GaAs. The results suggest that the increment observed in the gain is due to the larger dynamic range in InP than in n-GaAs films, where the optical scattering mechanisms play a drastic role rather than acoustic and ionized impurity scattering.

The study of microwave frequency conversion under negative differential conductivity will be one of the most relevant topics in microelectronics and communications in the coming years, due to the potential it represents in terms of amplification of microwave and millimeter-waves. However, in order to understand the behavior of non-stationary effects, a special attention must be paid to the transverse inhomogeneity, carrier-density fluctuations, in the plane of the film, because it may affect, in a negative way, the nonlinear wave interaction. Thus, a creation of effective algorithms and computer programs for simulations of non-linear interaction of space charge waves in semiconductor films, where the effects of non-locality and transverse inhomogeneity should be taken into account, becomes of high importance.

2. The equations for space charge waves

Consider n-InP film placed onto substrate without an acoustic contact. It is assumed that the electron gas is localized in the center of film. The thickness of the n-InP film is 2h < 1 μm, see Fig. 1. The coordinate system is chosen as follows: X-axis is directed perpendicular to the film, the electric field $E_{0}$ is applied along Z-axis, exciting and receiving antennas are parallel to Y-axis. 2D model of electron gas in the n-InP film is used. Thus, 2D electron concentration is presented only in the plane $x=0$. The space charge waves possessing phase velocity equal to drift velocity of the electrons $v_{0} = v(E_{0})$, $E_{0} = U_{0}/L_{z}$ are considered, where $U_{0}$ is bias voltage, $L_{z}$ is the length of the film. Generally, a non-local dependence of drift velocity $v_{0}$ of electrons on the electric field takes place.

In simulations, an approximation of two-dimensional electron gas is used. The set of balance equations for concentration, drift velocity, and the averaged energy to describe the dynamics of space charge waves within the n-InP film takes a form, like in GaAs film [7]:

$$\frac{dm(w)}{dt} = -q E \frac{\partial \phi}{\partial x}$$

$$\frac{dm}{dt} + \frac{\partial}{\partial x} [\phi \frac{\partial \phi}{\partial x}] = 0$$

$$D(w) \frac{D}{dt} \frac{w}{m(w)} = \frac{1}{2} \frac{D}{dt} \frac{1 - w^2}{2}$$

$$\bar{\phi} = \epsilon_{0} E_{0} = \epsilon_{0} \frac{q}{\partial x} \frac{1}{n_{0}} \frac{\partial n}{\partial x}$$

where $v_{d}$ is drift velocity, $\phi$ is the of potential, $n = n_{0} + \tilde{n}$ where $n_{0}$ is the equilibrium value of two dimensional concentration of electron gas, $\tilde{n}$ is the varying part, $w$ is the electron energy, $D$ is the diffusion coefficient, and $\epsilon_{0}$ is the lattice dielectric permittivity of n-InP, $m(w)$ is averaged effective mass, $q$ is the electron charge, $\tau_{p}(w)$ are relaxation times, and $E_{0}$ is the bias electric field. It should be noted that the delta function in equation Poisson implicitly defines the density which is important to correctly interpret the equation in actual physical quantities. It is assumed that a condition of occurring negative differential conductivity is realized. Because the signal frequencies are in microwave or millimeter wave range, it is possible to separate diffusion and drift motions. For a sake of simplicity, instead of relaxation times, the parameter $\tau_{p}$ is introduced [7]:

$$\frac{m(w)}{\tau_{p}(w)} = \frac{E_{1}}{v_{0}(E_{1})} w - w_{0} = qE_{1}v_{0}(E_{1})$$

In such a representation, the mean energy and effective mass of electron are denoted by $w$ and $m(w)$, the equilibrium value of $w$ is $w_{0}$. A direct correspondence between local field dependence and non-local effects is well seen. Because a dependence $E_{1} = E_{1}(w)$ is unique, it is possible to express the parameters $w$ and $v_{0}$ through the value of $E_{1}$. The dependences of drift velocity, averaged electron energy and averaged electron mass on electric field in InP films are taken from our Monte Carlo simulation results (see Fig. 2), which are in good agreement with measured data and the set of parameters used in the simulations is similar to the set published in [14]. The maximum value of drift velocity in InP is when the electric field is 10 kV/cm, under these high-field conditions some of the electrons can acquire enough energy from the electric field to transfer via inter-valley scattering to the upper valleys. Since the mobility of electrons in the upper valleys is smaller than in lowest valley, the conductivity decreases with increasing field, leading to a negative differential resistance (see Fig. 2a). A comparison of the results for GaAs and InP films reveals that for the field strength considered most of the electrons in GaAs are in higher valley while in InP less than are transferred. This is due to the difference in the separation of the valleys (0.3 eV in GaAs and 0.6 eV in InP) in spite of the higher field applied in InP [16]. A negative differential conductance appears when electric field is above the threshold field. Below that, the electrons do not have enough energy for inter-valley transfer. Above that, the intervalley-scattering and the optical scattering mechanisms are dominating, rather than acoustic and ionized impurity scattering. Since this process is inelastic, the electron energy curves have variation in their slopes: the average energy does not increase as fast with field as at lower fields (see Fig. 2b).

A small microwave electric signal $E_{ext}$ is applied to the input antenna. When this signal is applied, the excitation of space charge waves in 2D electron gas takes place. These waves are subject to amplification, due to negative differential conductivity.

The set of equations (1) form a set of non-linear coupled time dependent partial differential equations. These differential equations are discretized, using a finite-difference scheme and is solved numerically. A transverse inhomogeneity of the structure in the plane of the film along Y axis is taken into account. The following parameters are chosen: electron density in the film $n_{0} = 5 \times 10^{14}$ cm$^{-2}$, the initial uniform drift velocity of electrons is $v_{0} = 2 \times 10^{7}$ cm/s, the length of the film is $L_{z} = 0.1$ mm, the thickness of the film is $2h = 0.1–1$ μm.

3. Spatial increment of space charge waves

In this section, a description of propagation of longitudinal space charge waves in a negative differential conductance medium, using the drift–diffusion equations to find the dispersion relation, is presented; the equation of continuity (3) and Poisson’s equation (4) for electrons is

$$\frac{\partial m}{\partial t} + \frac{\partial}{\partial x} \left[ \nu \left( E_{1} - E_{0} \right) n - D \frac{\partial m}{\partial x} \right] = 0$$

where $m(w)$ is the particle mass, $\nu$ is the carrier mobility, $E_{1}$ is the bias electric field, and $D$ is the diffusion coefficient.
Fig. 2. Electron drift velocity (a), average electron energy (b), averaged mass (c) versus electric field used in simulations; (-) MC data, (○) Fischetti [14], and (□) Gonzalez [15] and experimental data: (△) Boers [8], (⋆) Glover [9], (□) Hamilton [10], and (△) Windhorn [11].

\[ n \frac{\partial n}{\partial t} + \frac{\partial}{\partial z} \left[ n \nu \tilde{n} + \frac{d
u}{dE} n_0 \tilde{E}_z - D \frac{\partial \tilde{n}}{\partial z} \right] = 0 \]  

(5)

\[ \nabla \tilde{\varphi} = \frac{|e|}{\varepsilon_0 \varepsilon} \tilde{n} \delta(x) \]  

(4)

where \( n \) is the surface concentration, \( n_0 \) is the equilibrium density, \( D \) is the diffusion constant and \( \tilde{E}_z = -\frac{\partial \varphi}{\partial z} \). Let us introduce \( \tilde{n} = n - n_0 \), \( \varphi = \varphi_0 + \tilde{\varphi} \) and \( E = \tilde{E}_0 + \tilde{E}_z \), with \( \tilde{E}_0 \) equal to the dc applied field, and \( \tilde{E}_z \) the rf field, with some approximations and mathematical tools the Eqs. (3) and (4) can be written as

\[ \frac{\partial \tilde{n}}{\partial t} + \frac{\partial}{\partial z} \left[ \nu \tilde{n} + \frac{d
u}{dE} n_0 \tilde{E}_z - D \frac{\partial \tilde{n}}{\partial z} \right] = 0 \]  

(5)

\[ \nabla \tilde{\varphi} = \frac{|e|}{\varepsilon_0 \varepsilon} \tilde{n} \delta(x) \]  

(6)

If one assumes that \( \tilde{n} = \tilde{N} \exp(-i\omega t + ikz) \) and \( \tilde{\varphi} = \phi(x) \exp(-i\omega t + ikz) \), then

\[ \tilde{N}(-i\omega + ikv_0 + Dk^2) + i k \frac{d\nu}{dE} n_0 \tilde{E}_z \big|_{x=0} = 0 \]  

(7)

\[ \frac{d^2 \phi}{dx^2} - k^2 \phi = -\frac{|e|}{\varepsilon_0 \varepsilon} \tilde{N} \delta(x) \]  

(8)

The Eq. (8) is equivalent to:

\[ \frac{d^2 \phi}{dx^2} - k^2 \phi = 0 \text{ for } x \neq 0 \]  

(9)

\[ \frac{d\phi}{dx} \big|_{x=0} - \frac{d\phi}{dx} \big|_{x=0} = -\frac{|e|}{\varepsilon_0 \varepsilon} \tilde{N} \phi \text{ is continuous at } x = 0 \]  

(10)

A proposed solution is

\[ \phi = \begin{cases} \frac{A}{B} \exp(-kx) & \text{for } x > 0 \\ \frac{A}{B} \exp(kx) & \text{for } x > 0 \end{cases} \]  

(11)

If \( \phi \big|_{x=0} = \phi \big|_{x=0} \), then \( A = B \) and

\[ \tilde{\varphi} = \frac{|e|}{\varepsilon_0 \varepsilon} \tilde{N} 2k \exp(-k|x|) \]  

(12)

then

\[ \tilde{E}_z \big|_{x=0} = -ik \tilde{\varphi} \big|_{x=0} = -\frac{|e|}{2\varepsilon_0 \varepsilon} \tilde{N} \tilde{n} \]  

(13)

If one substitutes the Eq. (13) into the Eq. (7) one obtains

\[ \tilde{N}(-i\omega + ikv_0 + Dk^2) + i k \frac{d\nu}{dE} n_0 + \left(-i\right)\frac{|e|}{2\varepsilon_0 \varepsilon} \tilde{N} = 0 \]  

(14)

Fig. 3. Spatial increments of instability \( k'(j) \) of space charge waves. Curve 1 shows results for an n-GaAs film [13]. Curve 2 is for n-InP films with \( E_0 = 20 \text{kV/cm} \), \( n_0 = 0.8 \times 10^{14} \text{ cm}^{-2} \). In curve 3, the electron density is \( n_0 = 5 \times 10^{14} \text{ cm}^{-2} \) with the same electric field and film thickness \( 2h = 0.05 \mu \text{m} \).

Fig. 4. Spectral components of the electric field of space charge waves. The effective excitation of harmonics is presented. The input carrier frequency is \( f = 12 \text{ GHz} \).
or
\[ \omega = \nu_0 k - i D k^2 - i \frac{|e| n_0}{2 \varepsilon_0 \varepsilon} \frac{d\nu}{dE} k \] (15)

Using the drift–diffusion equation, a study about of how a small, periodic disturbance may propagate in this InP film has been introduced by means the dispersion equation \( D(\omega, k) = 0 \), because it determines the modes of propagation, their phase velocities and group velocities and also show if any instability can exists. For our present purpose, which is when the negative differential conductivity shows up, \( d\nu/dE < 0 \), space charge waves will get amplified instead of being damped. In general, we consider the cases where \( \omega = 2\pi f \) is real and \( k = k' + ik'' \) has real and imaginary part. The case \( k' > 0 \) corresponds to spatial increment (amplification), whereas the case \( k' < 0 \) corresponds to the decrement (damping). In Fig. 3, the spatial increment of space charge waves in an n-InP film is shown in the curve 2, where the electron concentration is \( n_0 = 0.8 \times 10^{14} \text{ cm}^{-2} \), the bias electric field is \( E_0 = 20 \text{kV/cm} \).

In curve 3, the electron density is \( n_0 = 5 \times 10^{14} \text{ cm}^{-2} \) with the same bias electric field. Curve 1 is the result for n-GaAs films where the electron density is \( n_0 = 5 \times 10^{14} \text{ cm}^{-2} \) and the bias electric field is \( E_0 = 4.5 \text{kV/cm} \). The stationary values of \( E_0 \) have been chosen in the regime of negative differential conductivity \( (d\nu/dE < 0) \) for all cases. It can be seen that an amplification of space charge waves in InP films occurs in a wide frequency range, and the maximal spatial increment is \( k'' = 3 \times 10^5 \text{ m}^{-1} \) at the frequency \( f = 35 \text{ GHz} \). When compared with a case of the GaAs film, it is possible to observe an amplification of space charge waves in InP films at essentially higher frequencies \( f > 44 \text{ GHz} \). To obtain an amplification of 25 dB, it is necessary to use a distance between the input and output antennas of about 0.09 mm.

4. Results and simulation

The propagation and amplification of space charge waves in n-GaAs thin films with negative difference conductance have been

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Fig. 5. The spatial distributions of the alternative part of the electric field component \( E_z \), the component of electric field \( E_z \), alternative part of the electron concentration \( n^− \), and the component of the electron drift velocity \( n^− \), the length of the film is 0.1 mm. The transverse width of the film along \( Y \) axis is 1 mm.
studied in the last decade [12,13], however n-InP films have not been addressed yet, and are subject of this work. We address the device presented in Fig. 1 by means of numerical simulations. An n-InP epilayer film of thickness 0.1–1 μm is put on an InP semi-insulating substrate. The two-dimensional electron density in the film is chosen to be \( n_d = 5 \times 10^{14} \text{ cm}^{-2} \). On the film surface are the cathode and anode ohmic contacts (OCs), together with the input and output coupling elements (CEs). Designed as a Schottky-barrier strip contacts, the CEs connect the sample structure to microwave sources. A dc bias voltage (above the Gunn threshold, 20 kV/cm) was applied between the cathode and anode OCs, causing negative differential conductivity in the film. The CEs perform the conversion between electromagnetic waves and space charge waves, where the excitation of space charge waves in the 2D electron gas takes place. In the simulations an approximation of two-dimensional electron gas is used.

A small microwave electric signal

\[ E_{\text{ext}} = E_0 \sin(\omega t) \exp(-(z-z_1)/L_z^2 - (x-x_1)/L_x^2) \]

is applied to the input antenna. Here \( z_1 \) is the position of the input antenna, \( z_0 = 0.1 \) μm. The carrier frequency \( f = 0.1 \) GHz. When a small microwave signal is applied to the input antenna, the excitation of space charge waves in the 2D electron gas takes place. The space charge waves are subject to amplification, due to the negative differential conductivity. The stable implicit difference scheme is used. The following parameters have been chosen: electron density in the film is \( n_d = 5 \times 10^{14} \text{ cm}^{-2} \), the initial uniform drift velocity of electrons is \( v_0 \), the length of the film is \( L_z = 0.1 \) μm, the thickness of the film is \( 2h = 0.1 \) μm. The typical output spectrum of the electromagnetic signal is shown in Fig. 4. The input carrier frequency is \( f = 12 \) GHz. The amplitude of the input electric microwave signal is \( E_m = 25 \) V/cm. Although the growth rate decreases as the \( f \) frequency increases, for our case an amplification of 25 dB is obtained. The duration of the input pulse is \( t_{\text{ip}} = 25 \) ns. The maximum of the input pulse occurs at \( t_1 = 2.5 \) ns. One can see both the amplified signal at the first harmonic of the input signal and the harmonics generations of the input signal, which is generated due to the non-linearity of space charge waves. The spatial distributions of the alternate component of the electric field \( E_{x_1}, E_{x_2} \) are shown in Fig. 5, at the time moment \( t = 4 \) ns. One can see the maximum variations are in the output antenna and the spatial distribution in direction \( E_x \) is bigger than in \( E_y \), because it is the propagation of space charge wave. The length of the film is 0.1 mm. The transverse width of the film along \( Y \) axis is 1 mm. The duration of the input electric pulse is 2.5 ns. The spatial distributions are presented for the time moment 1.5 ns after the maximal value of the input signal. Direct numerical simulations have confirmed pointed below results on linear increments of space charge waves amplification. Also a possibility of non-linear frequency doubling and mixing is demonstrated. To get the effective frequency doubling in the millimeter wave range, it is better to use the films with uniform doping.

5. Conclusions

A theoretical study of two-dimensional amplification and propagation of space charge waves in n-InP films is presented. A microwave frequency conversion using the negative differential conductivity phenomenon is carried out when the harmonics of the input signal are generated. A comparison of the calculated spatial increment of instability of space charge waves in n-GaAs and n-InP films is performed. An increment in the amplification is observed in InP films at essentially higher frequencies \( f > 44 \) GHz than in GaAs films, which is due to its larger dynamic range. The maximum amplification (gain of 25 dB) is obtained at \( f = 35 \) GHz, using a distance between the input and output antennas of about 0.09 mm.

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References


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Vassili Palankovski was born in Sofia, Bulgaria, in 1969. He received the diploma degree in electronics from the Technical University of Sofia in 1993. Afterwards he worked for three years in the telecommunications field. In March 1997, he joined the Institute for Microelectronics at the Technische Universität Wien, where he received his doctoral degree in technical sciences in 2000 and continued as a post-doctoral researcher. In summer 2000, he held a visiting research position at LSI Logic Corporation, Milpitas, CA. In 2003, Dr. Palankovski and Dr. Quay published the book Analysis and simulation of heterostructure devices in the Springer series on computational microelectronics. In 2004, he joined Infinion Technologies, Villach, Austria, for half a year as a technology development engineer. Having received the highest Austrian award for young scientists (START-Prize), Dr. Palankovski returned to the Technische Universität Wien in 2005 to establish the Advanced Materials and Device Analysis group. In 2008, he was elected a member of the young curia of the Austrian Academy of Sciences.