Electromagnetic Coherent Electron Control

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Abstract—Electron quantum optics offers fascinating insights into the dynamic electron evolution processes governed by quantum effects, attractive for novel electronic processing or sensing devices. A key requirement for these developments is to coherently and electromagnetically confine and control the electron evolution process and the ability to correctly describe the manifesting quantum effects related to the wave nature of the electron, e.g., interference. This work provides an overview of research conducted on using specifically shaped electric and magnetic fields to influence the electron evolution in nanostructures. The Wigner based quantum transport modeling approach is used to simulate the transport and to highlight quantum effects.

Index Terms—Electron quantum optics, Single-electron electronics, Wigner transport equation, Electron quantum transport, Electromagnetic fields, Quantum interference, Entangletronics

I. Introduction

The astonishing developments in electronics led to continuous reductions in feature sizes over the past decades, a process which is still ongoing. Significant efforts are devoted to evolve electronics into the single-electron regime enabled by substantial nanotechnological advancements, promising reduced power consumption and even higher integration densities [1]. A first important stepping stone was single-electron electronics [2], which, aside from introducing the single-electron tunneling transistor [3], also yielded the first single-electron sources, focusing on charge transfer [4]. Later, the single-electron source has been further advanced towards generating *coherent* single electrons [5] and is still advanced today (e.g. [6]): The generated electrons have well-defined wave-functions (e.g. Gaussian form [7]), enabling to engineer coherent manipulations of electrons similar as in the optical world. In addition to advanced electron sources, the ability to confine and control single electrons in nanostructures [8] and to characterize quantum-coherent circuits [9] further advanced the field of coherent single-electron devices and circuits. Ultimately, these developments gave rise to the field of electron quantum optics [10][11][12][13][14].

Making use of the wave nature of electrons proved vital for many fields of applications, such as quantum information processing (e.g. flying electron qubits [8][15]), quantum sensing [16], and quantum metrology [17]. Although facing significant challenges with shorter coherence times compared to photonic approaches, the ability to fabricate single-electron quantum circuits based on advanced but well-understood solid-state technologies with sub-micron to single-digit nanometer feature scales offers the potential for very high integration densities and large-scale mass production via today's well-matured semiconductor electronics industries. As such, quantum circuits offer an attractive path towards efforts regarding *Beyond CMOS* and solid-state based quantum information processing in general [1].

As hinted previously, what is essential to all of the above advancements is the ability to coherently control the electron evolution (for a review see [8]). Typical approaches are based on quantum dots [18] and nanotubes [19] whereas other approaches are based on quantum point contacts with magnetic [20] or electrostatic focusing [21]. Related to these findings, another approach to coherently control electrons, aiming at advanced logic devices and systems as part of our efforts in entangletronics [22][23], is to control the electron coherence to realize quantum interference devices. We present quantum transport simulations in phase space to predict the involved physical phenomena (see Section II). This control is motivated by Young-like double-slit structures and Aharonov-Bohm rings; both are fundamental to control the interference pattern of electrons. Alternatively and at the center of our research, electron control can be established by specifically shaped electric and magnetic fields, which can manipulate the state of a single electron in specific ways.

In the following, we first summarize our modeling approach, which is followed by discussing key findings regarding electric and magnetic coherent control of electrons.

II. Modeling Approach

We use a signed-particle Wigner transport model [24], implemented in VIENNAWD[1] [25][26], to stochastically describe the dynamic quantum transport processes of individual electrons in two-dimensional devices and structures. In general, the underlying Wigner function has found broad application in science and engineering [27], in particular in recent years [28], due to its unique properties: The Wigner function f_W is a real function, which can have negative values, but retains the basic properties of the classical statistical distribution; the Wigner function is thus referred to as a *quasi-distribution* function. Physical averages can be obtained from the Wigner function in the same way as in classical statistics.

[1]www.iue.tuwien.ac.at/software/viennawd/

In particular, the most important property of the Wigner function is that the mean value of a physical quantity A is given by

$$\langle A(t) \rangle = \int \int A(\boldsymbol{p}, \boldsymbol{r}) f_W(\boldsymbol{p}, \boldsymbol{r}, t) \mathrm{d}\boldsymbol{p} \mathrm{d}\boldsymbol{r} , \qquad (1)$$

where $\boldsymbol{p}$ is the momentum, $\boldsymbol{r}$ the position $((\boldsymbol{p}, \boldsymbol{r})$ spans the phase space), and t the time.

The Wigner function is modeled by stochastic numerical particles, which evolve in phase space, bearing most of the properties of the classical particle model (Boltzmann transport), like Newtonian trajectories and ensemble averaging. However, additional properties and mechanisms, such as particle sign and evolution rules (i.e. particle generation and annihilation), are introduced to account for the quantum information in the system.

At the core of modeling quantum electron evolution is the equation of motion for the Wigner function (f_W) – the Wigner transport equation [29][30].

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{m} \cdot \frac{\partial}{\partial \boldsymbol{r}} + e \frac{\boldsymbol{p}}{m} \times \boldsymbol{B} \cdot \frac{\partial}{\partial \boldsymbol{p}} \right] f_W(\boldsymbol{p}, \boldsymbol{r}, t) = $$
$$\int d\boldsymbol{p}' V_W(\boldsymbol{p} - \boldsymbol{p}', \boldsymbol{r}) f_W(\boldsymbol{p}', \boldsymbol{r}, t) \qquad (2)$$

$\boldsymbol{B}$ is the magnetic field, m the effective electron mass, e the elementary charge, and V_W the Wigner potential. The electric component $\boldsymbol{E}$ of the Lorentz force is embodied in the Wigner potential V_W, allowing to describe quantum effects (e.g. non-locality) via higher order derivatives of the classical electric potential. Therein lies an attractive, natural ability of a Wigner based transport modeling approach: Only considering the first derivative of the electric potential in the Wigner potential calculation allows to switch to a classical description – the Wigner transport equation reduces to the Boltzmann transport equation. Moreover, Wigner transport modeling allows to consider boundary conditions [31][32] and scattering processes [33].

In the here presented studies, an electron is modeled as a minimum uncertainty wave packet, which is described by a Wigner distribution representing an admissible Wigner pure state [34]

$$f_W(\boldsymbol{r}, \boldsymbol{k}) = N \exp\{ -\left| \boldsymbol{r} - \boldsymbol{r}_0 \right|^2 / (2\sigma^2) \} \exp\{ -\left| \boldsymbol{k} - \boldsymbol{k}_0 \right|^2 2\sigma^2 \} , \qquad (3)$$

where N is a normalization constant and the wave vector $\boldsymbol{k}$ is related to the momentum variable via $\boldsymbol{k} = \boldsymbol{p}/\hbar$. The minimum uncertainty wave-packet, with a standard deviation σ, is characterized by a Gaussian distribution of the momentum with constant variance determined by the variance of the corresponding components in the position space.

III. ELECTROSTATIC LENSE

A first investigation studied the influences of a specifically-shaped electric potential *lense* (i.e. a potential distribution which focuses the electron) on the electron evolution including phonon scattering in a $200 \times 120 \, \mathrm{nm}^2$ structure [22]. To fully investigate coherent transport, a deeper understanding of the scattering-induced transition to classical transport is essential.
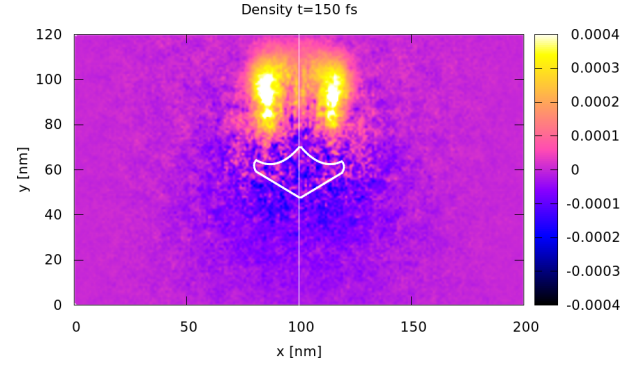

Fig. 1: Difference between the coherent and phonon-affected electron densities [a.u.] after 150 fs evolution for a specifically shaped electric lense and open boundary conditions. Reprinted with permission from Ellinghaus *et al.* [22]. © 2017 WILEY-VCH.

Both, coherent processes and scattering-caused transitions to classical dynamics were unified by a scattering-aware particle model of the lense-controlled state evolution. The approach bridges the theory of coherence with the Wigner signed-particle model. Fig. 1 shows the difference of the coherent and phonon-modified electron densities for the investigated electric lense (indicated by the white isoline). The regions where scattering dominates (i.e. negative difference) are colored in blue. The spread of the phonon-aware density is restricted as compared to the quantum counterpart, clearly demonstrating the scattering-induced localization.

IV. SINGLE BARRIER: CLASSICAL/QUANTUM TRANSPORT

In other work, an analysis of the quantum coherent processes involved in the electron evolution in a quantum wire $(20 \times 30 \, \mathrm{nm}^2)$ which hosts a centrally placed repulsive potential barrier was investigated [35]. The barrier is modeled by means of a screened Coulomb potential. Fig. 2 shows the classical and quantum current density with lateral reflecting boundary conditions and a peak barrier level of $0.35 \, \mathrm{eV}$, indicated by the circular isolines. In the quantum case the current density path is much more closed around the barrier than in the corresponding classical case, which is due to the joint action of non-locality, tunneling, and repulsion, including the influence of the boundary condition associated with the quantum wire.

V. SINGLE BARRIER: WIGNER FUNCTION NEGATIVITY

In follow up work, a unique feature of the Wigner function – the negativity – was further investigated with respect to highlighting quantum effects in electron quantum transport hosting a repulsive Coulomb potential barrier [36]. Analyzing Wigner function negativity and establishing relationships to quantum effects is widely applied in other fields [27][28]. The quantum distribution reveals a clear negative part *behind* the potential barrier between the two correlated branches (Fig. 3). In contrast, the classical distribution does not show such negative excursions.

VI. DOUBLE WELLS: INTERFERENCE EFFECTS

An analysis of interference effects as a result of the electron evolution within a coherent transport medium hosting two Coulomb potential wells (i.e. *attractive*) was investigated [37].
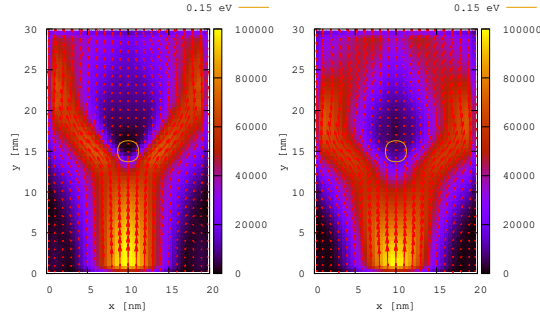
Fig. 2: Classical (left) and quantum (right) current density [a.u.] with lateral reflecting boundaries for a quantum wire and a centrally placed repulsive Coulomb potential barrier. Reprinted with permission from Ballicchia *et al.* [35]. © 2018 Authors, licensed under the Creative Commons Attribution 3.0 Unported License.
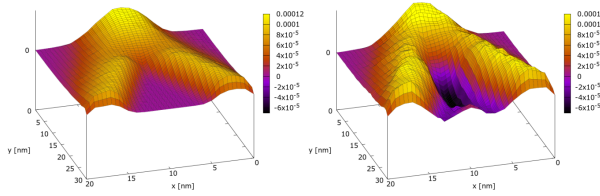

Fig. 3: Classical (left) and quantum (right) sum of Wigner phase space distributions taken from representative locations in the momentum sub-space. A rotated viewport relative to Fig. 2 is used. Reprinted with permission from Ballicchia *et al.* [36]. © 2019 Authors, licensed under the Creative Commons Attribution 4.0 International License.

The introduction of an additional potential well introduces new quantum effects caused by the non-locality of the action of the quantum potential, leading to pronounced interference effects. Fig. 4 shows the electron density for all absorbing boundary conditions in the classical and in the quantum case. The green isolines indicate the location of the wells. In the classical case, no interference pattern can be recognized beyond the wells as the action of the electric force is local. The effect of the two potential wells is, however, noticeable behind the wells: Right behind each well, the symmetry of the density distribution reflects the symmetry of the Coulomb force. In the quantum case, the non-locality action of the quantum potential of the wells affects the injected electrons right after injection. In the lower part, the density follows the symmetry of the individual Coulomb potentials: Two channel-like density maxima are formed. In the upper half and fundamentally different to the classical case, an intricate interference pattern manifests, establishing a similarity to double-slit experiments.

VII. DOUBLE WELLS: MAGNETIC FIELD

In follow up work, the influence of a uniform magnetic field on the evolution process was investigated [38][39], requiring a full electromagnetic description. Fig. 5 shows Wigner function negativity maps for an asymmetric well configuration and for the symmetric case with an applied magnetic field.
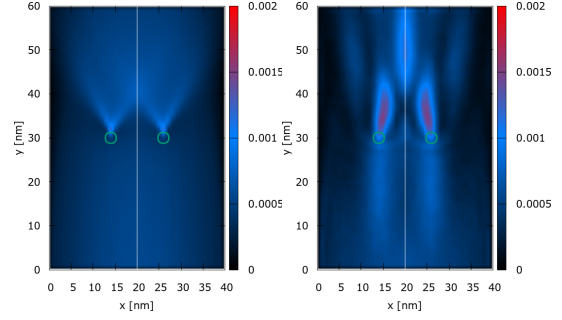

Fig. 4: Classical (left) and quantum (right) electron density [a.u.] with lateral absorbing boundaries for a quantum wire and two centrally placed dopant potentials (0.365eV). Reprinted with permission from Weinbub *et al.* [37]. © 2018 Authors, licensed under the Creative Commons Attribution 4.0 International License.
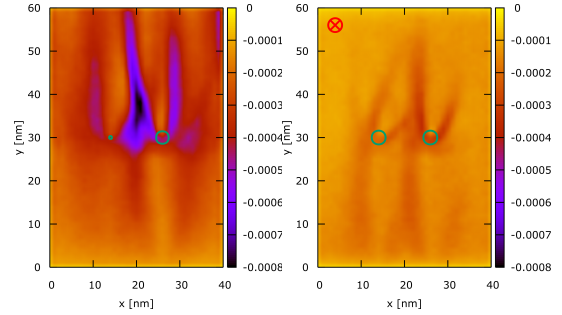

Fig. 5: Wigner function negativity map for asymmetric potential wells and no magnetic field (left) and symmetric potential wells and applied magnetic field (right); green isolines indicate potential wells. Reprinted with permission from Ferry *et al.* [38]. © 2020 Authors, licensed under the Creative Commons Attribution 4.0 International License.

The results reveal three important conclusions: (1) reducing the left well's peak potential level pushes the interference pattern to the right; (2) the magnetic field has a similar effect; (3) contrary to the pure electric field based manipulation, the influence of the magnetic field significantly reduces the negativity of the Wigner function, indicating a loss of coherence.

VIII. SUMMARY

The Wigner based quantum transport modeling approach allows to predict dynamic quantum and classical transport processes, enabling to investigate the transition from the quantum coherent domain to the scattering-induced classical domain. In addition, the ability to investigate the negativity of the Wigner function provides yet another tool to identify quantum effects. As was shown, specifically shaped potential barriers or wells allow to design the electron transport and by extension the arising quantum effects, most importantly the interference pattern. Although using magnetic fields provides an alternative way of electron control, the negativity and as such the coherence is destroyed as well.

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