A Physics-Based Impact Ionization Model Using Six Moments of the Boltzmann Transport Equation

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ABSTRACT

Due to the ever decreasing device geometries nonlocal effects gain more and more importance. It is particularly well known that impact ionization is not properly described by neither a local field nor a local energy model because it is mainly determined by the high-energy tail of the carrier distribution function. Information about the high-energy tail is lost when only the average carrier energy is taken into account. To overcome this limitation, we use the fourth moment of the distribution function to account for the population of the high-energy tail. We propose a new impact ionization model using this tail temperature and compare the results obtained with existing models and Monte-Carlo simulations which show improvements obtained by our new model.

Keywords: Device Simulation, Impact Ionization, Moments Method, Boltzmann's Equation

1 INTRODUCTION

As device geometries are further reduced without according reduction of the supply voltages, the electric fields occurring inside the devices increase rapidly. Furthermore, strong gradients in the electric field are observed. These highly non-homogeneous field distributions give rise to distribution functions which deviate significantly from the frequently assumed Maxwellian distribution. Furthermore, as has been pointed out in [1], the distribution function (DF) is not uniquely described using just the average carrier energy. This is depicted in Fig. 1 which shows some electron distribution functions inside a channel of a MOS transistor. Points ABC are in the channel while the points D and E are taken from the drain region. In the drain region, the overpopulation of the high-energy tail is obvious, whereas in the channel it is underpopulated, showing a significant thermal tail [2].

Several moment based models have been proposed so far which aim at obtaining some additional information about the DF to the average energy. One approach is to split the energy range at some characteristic energy and handle both energy ranges with a two population and temperature model [3], [4]. As these models were aimed at modeling of impact ionization (II) the band gap energy was taken as the characteristic energy. This approach leads to various additional scattering rates which model the transitions between the two energy regions relying on carefully set up Monte-Carlo (MC) simulations. Due to this specialization to II, this model would have to be reformulated if another energy range is of interest as is the case for the calculation of gate currents. Thus this approach is difficult to generalize if both effects need to be captured at the same time which is demanded for state-of-the-art devices. A special formulation using two electron populations has been proposed in [5] for those regions where the high-energy tail is heavily populated.

2 SIX MOMENT METHOD

Several authors gave higher order moment equations to obtain additional accuracy, see e.g., [6], [7]. These equations were based on an Ansatz for the DF which was taken to be some expansion around a Maxwellian distribution. Sonoda *et al.* [8] added two equations for the fourth and fifths moment of BTE to a standard energy-transport model taken from [9]. We follow a similar approach and use six moments to model II but use a second temperature

$$\Theta_\nu = \frac{2}{5\,\mathrm{k_B}}\,\frac{\langle\mathcal{E}^2\rangle}{\langle\mathcal{E}\rangle} = \frac{4}{15}\,\frac{\langle\mathcal{E}^2\rangle}{\nu\,\mathrm{k_B^2}\,T_\nu} \tag{1}$$

as a new solution variable with ν being the carrier type (n or p). For a Maxwellian distribution $\Theta_\nu = T_\nu$ and thus any deviation of Θ_ν from T_ν indicates a deviation from the Maxwellian distribution. This approach has the advantage that it does not depend on any splitting energy. Just two additional relaxation times are needed.

3 PREVIOUS II MODELS

A commonly used assumption [10] for modeling II is that the ionization rates depend on the local carrier temperature T_ν in the following way

$$G_{\nu_{ii}} = \nu\,g_{ii}\,\exp\Big(-\frac{\mathcal{E}_C}{k_B\,T_\nu}\Big) \tag{2}$$

with g_{ii} and $\mathcal{E}_C$ being fit-factors which strongly depend on the technology and device geometry. Although a lo-

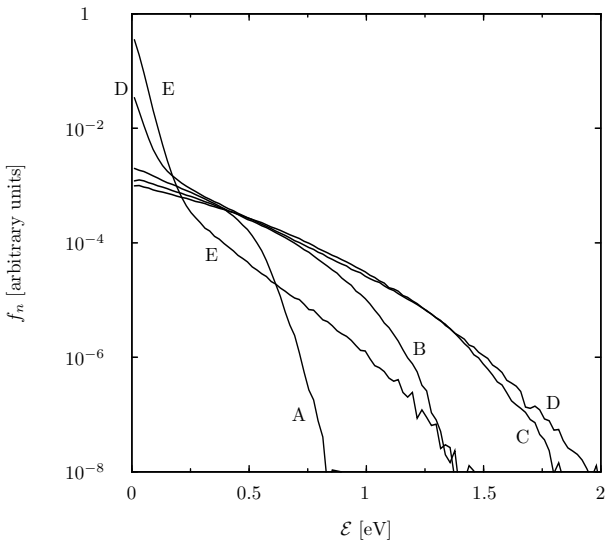

Figure 1: Electron distribution function at five characteristics points inside the channel of a MOS transistor. Note that the energies for the points A and D are the same.

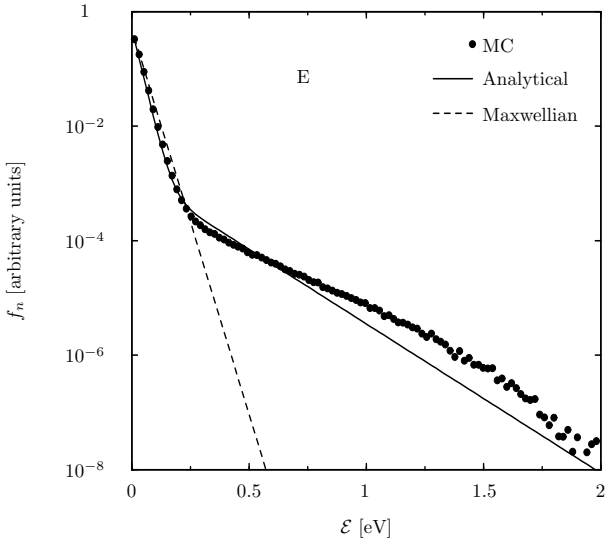

Figure 2: Distribution function at point E with the analytical model in comparison to MC results. Also shown is the Maxwellian using the average carrier energy.

cal energy model (LE) is capable of reproducing measured integral quantities like contact currents, the predicted ionization rates *inside* the devices have been shown to deviate significantly from MC simulations. This dramatically limits the usefulness of the simulations and makes predictive device simulations nearly impossible. To overcome the limitations of the above expression [8] derived an expression for the II coefficients using six moments of the DF by fitting the following analytical expression to their MC simulations

$$G_{\nu_{ii}} = \nu\, g_{ii}\, \exp\Big(-\frac{\mathcal{E}_C}{\langle\mathcal{E}\rangle_{\text{eff}}}\Big) \tag{3}$$

$$\langle\mathcal{E}\rangle_{\text{eff}} = \langle\mathcal{E}\rangle\, \exp\Big(\gamma\,\Big(\sqrt{\frac{\Theta_\nu}{T_\nu}} - \xi_h\Big)\Big) \tag{4}$$

with g_{ii}, γ, and ξ_h being fit-factors.

4 NEW II MODEL

Instead of trying to find an appropriate fit expression we make use of the following observation: when hot carriers from the channel of a MOSFET enter the drain region, they mix with the large pool of cold carriers [5]. As only the hot carriers cause impact ionization we estimate the density and temperature of these hot carriers from the available quantities by assuming a superposition of two Maxwellian distributions f_{ν_0} and f_{ν_1} which has been shown in [5] to give an excellent agreement with MC simulation results. Under the assumption of parabolic bands and that the pool is at lattice temperature ($T_{\nu_0} = T_L$), expressions for the moments of f_{ν_1} can be derived which read

$$T_{\nu_1} = t_\nu\,(\Theta_\nu - T_L) \tag{5}$$

$$\frac{\nu_1}{\nu} = \frac{T_\nu}{T_\nu + t_\nu^2\,(\Theta_\nu - T_\nu)} \tag{6}$$

$$\text{with} \qquad t_\nu = \frac{T_\nu}{T_\nu - T_L} \tag{7}$$

T_{ν_1} and ν_1/ν approximate the temperature and population of the high-energy tail, respectively. The analytical expression for the distribution function is then

$$f = C\,\Big(\frac{\nu-\nu_1}{(\mathrm{k_B}\,T_L)^{3/2}}\,\exp\Big(-\frac{\mathcal{E}}{\mathrm{k_B}\,T_L}\Big) + \frac{\nu_1}{(\mathrm{k_B}\,T_{\nu_1})^{3/2}}\,\exp\Big(-\frac{\mathcal{E}}{\mathrm{k_B}\,T_{\nu_1}}\Big)\Big)$$

A comparison of the distribution function obtained this way at point E vs. MC results is shown in Fig. 2. Also shown is the Maxwellian distribution with average carrier energy $T_n = 374$ K from point E. As only 4 % of the carriers contribute to the high-energy tail, the average energy is mostly determined by the low-energy part of the distribution.

We now use T_{ν_1} and ν_1 to replace T_ν and ν in a simple local energy model to obtain the new II rates as

$$G_{\nu_{ii}} = \nu_1\, g_{ii}\, \exp\Big(-\frac{\mathcal{E}_C}{k_B\,T_{\nu_1}}\Big) \tag{8}$$

The two parameters have been derived from MC simulations as $g_{ii} = 2.16 \cdot 10^{13}\mathrm{s}^{-1}$ and $\mathcal{E}_C = 4$ eV.

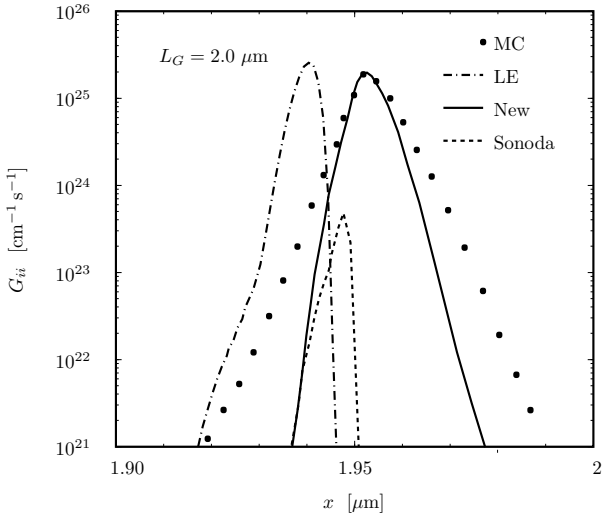

Figure 3: Comparison of the 3 models in a horizontal cut through the points of maximum generation in the MC simulations for the long-channel device.

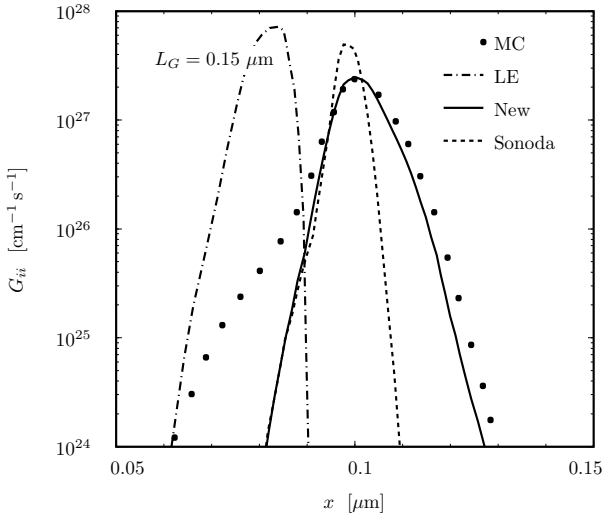

Figure 5: Comparison of the 3 models in a horizontal cut through the points of maximum generation in the MC simulations for the short-channel device.

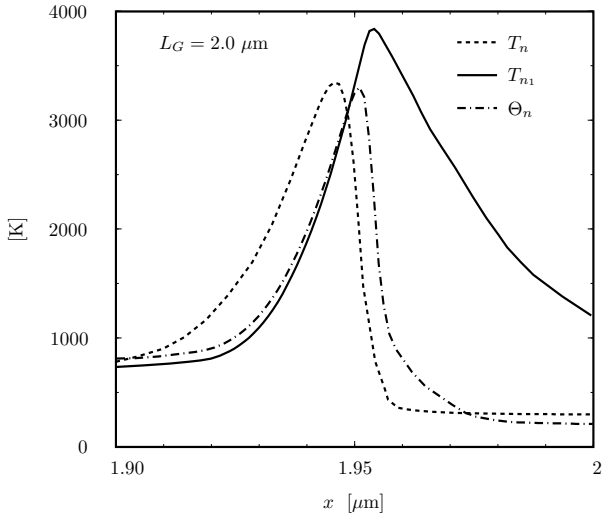

Figure 4: Electron tail temperature T_{n_1} together with T_n and Θ_n for the new model at the same cross-section for the long-channel device.

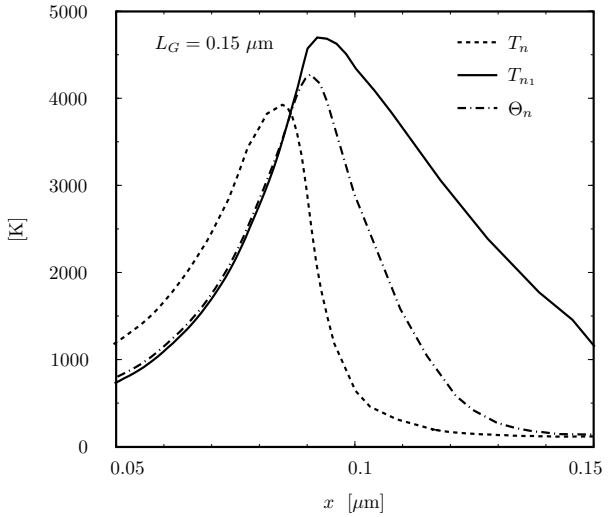

Figure 6: Electron tail temperature T_{n_1} together with T_n and Θ_n for the new model at the same cross-section for the short-channel device.

5 EXAMPLE AND DISCUSSION

As an example device we considered two MOSFETs with gate-lengths $L_G = 2.0$ μm and $L_G = 0.15$ μm. A comparison of the II rates of the new model to MC simulations, the simple LE model, and the model proposed by [8] is shown in Fig. 3 and Fig. 5 for a long-channel and a short-channel device, respectively. As expected, the LE model predicts the peak of G_{ii} at the wrong position whereas the position predicted by the models based on six moments agrees quite well with the MC data. However, the model proposed by [8] shows a too steep decline (as can also be seen in Fig. 3 in [1]) and does not properly model the long-channel device, at least not with the parameters given in [1]. In Fig. 4 and Fig. 6 the tail temperature T_{n_1} is shown together with T_n and Θ_n for the same cross-section. A comparison of the simulated substrate currents is given in Fig. 7 and Fig. 8.

For the prediction of device performance at even shorter gate lengths it is important for the model to be as physical as possible and to contain few fitting parameters which we believe is the strength of our new model. In addition to the better reproduction of the shape and location of the maximum of G_{ii}, our new model proved

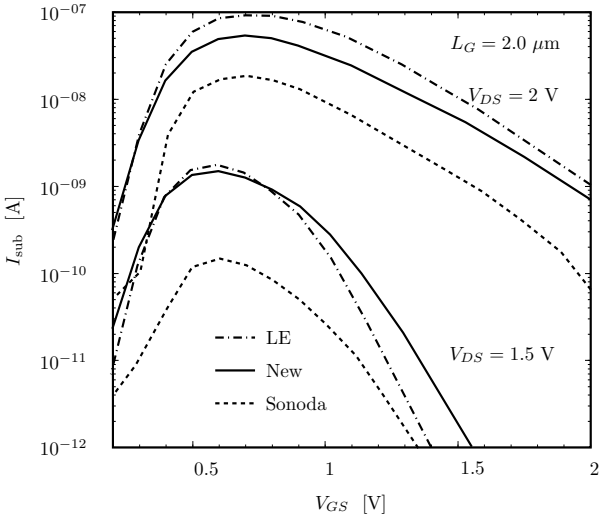

Figure 7: Comparison of the substrate currents delivered by the three models for the long-channel device.

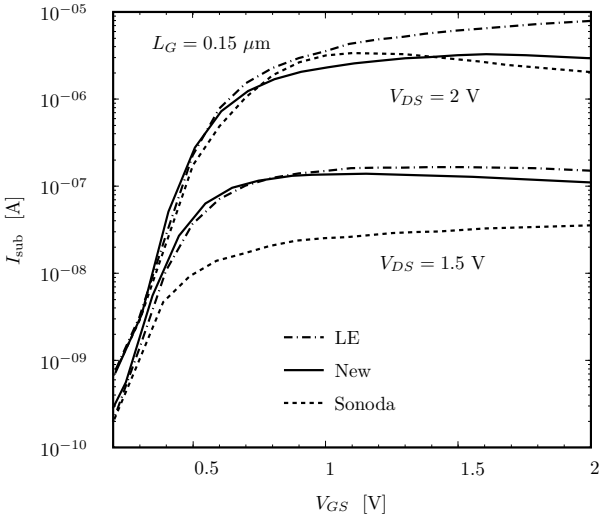

Figure 8: Comparison of the substrate currents delivered by the three models for the short-channel device.

to be numerically efficient and stable. The required simulation times of the new model exceeded those of the LE hydrodynamic model only by about 15 % compared to 100 % demanded by the Sonoda *et al.* model.

6 CONCLUSIONS

We have proposed a new impact ionization model based on a six moments transport description. A comparison of our new method with existing approaches outlines the qualities of our model. Despite the physically motivated expression, the model also proved to be numerically very efficient compared to existing approaches. Due to the good agreement with MC simulations we believe that the necessary two parameters are valid over a wide range which makes our new model a very good choice to accurately predict II effects for deep submicron devices.

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