Closed-Form Electrostatic Field Analysis for Metallic Comb-like Structures Containing Single and Interconnected Floating Strips of Arbitrary Topological Comlexity
Part II: Two-Dimensional Representation

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I. Introduction

In recent years the design and technology of surface acoustic wave (SAW) interdigital transducers have achieved such a high standard, that for some cases of greatest practical interest the one-dimensional representation and analysis of SAW filters cannot fulfill the imposed severe requirements. Therefore, the calculation of the two-dimensional (2D) field distribution has become indispensible, especially for accurate determination of SAW diffration pattern due to small finger overlap. Using the 2D Green's function in the wavenumber domain, $\overline{G}(k_x, k_z)$, the spectral domain representation and the moment method, we have developed a non-iterative semi-numerical method for the 2D electrostatic field analysis of arbitrary thin metallic structures (fingers) deposited on the surface of an anisotropic semi-infinite dielectric. Recently we have shown that $\overline{G}(k_x, k_z)$ can be expressed in closed form [1]. Here we will review the properties of $\overline{G}(k_x, k_z)$ and show that a natural extension of the one-dimensional concept of part I can successfully be applied to the 2D electrostatic field problem. First, an integral representation is established and then the associated integral equation is replaced by a matrix equation. The elements of the corresponding matrix, a modified inverse capacitance matrix, are calculated in closed form. An efficient non-equidistant discretisation of the metallic fingers has been chosen to take into account the edge-singularities of the fingers. The floating fingers, which may be of arbitrary topological complexity, are easily included in the analysis. Since the Green's function does not depend on a special finger geometry, for a given crystal-cut it is possible to deduce from $\overline{G}(k_x, k_z)$ characteristic functions, which describe the crystal anisotropic effects of the substrate and can conveniently be tabulated. Together with the fact that $\overline{G}(k_x, k_z)$ and the elements of the inverse capacitance matrix are closed-form formulae, the influences of the substrate anisotropy and of the finger geometry on the field distribution can be investigated separately and efficiently. An independent, critical and reasoned account of the subject is presented.

Among others, the transverse end-effects will be discussed.

II. Theory

Consider a semi-infinite anisotropic dielectric characterized by $(\underline{\epsilon})$. $(\underline{\epsilon})$ is a three by three symmetric positive definite matrix [1], [2], i.e.

$$(\underline{\epsilon}) = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

$$\epsilon_{11} > 0,\ \epsilon_{22} > 0,\ \epsilon_{33} > 0$$

$$\epsilon_{11}\epsilon_{22} - {\epsilon_{12}}^2 > 0,\ \epsilon_{11}\epsilon_{22} - {\epsilon_{12}}^2 > 0,\ \epsilon_{22}\epsilon_{33} - {\epsilon_{23}}^2 > 0$$

$$det(\underline{\epsilon}) > 0.$$

A set of infinitely thin electrodes (fingers) may be deposited on the plane surface of the substrate. There are no restrictions imposed on the 2D-geometry of the fingers and on the finger potentials. The finger are assumed to have negligible sheet resistivity, Fig.1.

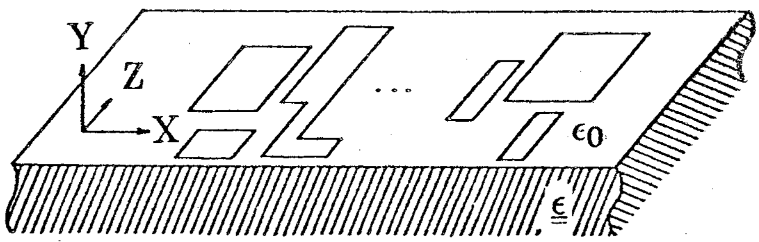

Fig.1: Two dimensional representation of a SAW transducer

With regard to the above conditions and assuming a time variation as $e^{j\omega t}$, the charge density $\rho(x,z)$ and the potential $\Phi(x,z)$ on the surface (y=0) are related by a convolution equation involving Green's function $G(x,z)$

$$\Phi(x,z) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(x\prime - x, z\prime - z)\rho(x\prime, z\prime)dx\prime dz\prime + C \qquad (II.1)$$

The physical meaning of the constant parameter C will be discussed in a later section. The Fourier transform of (II.1) is

$$\overline{\phi}(k_x,k_z) = \overline{G}(k_x,k_z)\cdot\overline{\rho}(k_x,k_z) + C\cdot\overline{\delta}(k_x,k_z) \qquad (II.2)$$

Recently an analytical formula for $\overline{G}(k_x,k_z)$ has been published [2], where the properties of $\overline{G}(k_x,k_z)$ extensively are discussed. $\overline{G}(k_x,k_z)$ has the form

$$\overline{G}(k_x,k_z)=\frac{\Gamma}{\sqrt{k_x^2+k_z^2+\sqrt{\alpha k_x^2+2\beta k_x k_z+\gamma k_z^2}}} \qquad (II.3-a)$$

wherein $\Gamma=\frac{1}{\epsilon_0}$ and

$$\alpha=\epsilon_{11}\epsilon_{22}-\epsilon_{12}^2 \qquad (II.3-b)$$

$$\beta=\epsilon_{13}\epsilon_{22}-\epsilon_{12}\epsilon_{23} \qquad (II.3-c)$$

$$\gamma=\epsilon_{22}\epsilon_{33}-\epsilon_{23}^2 \qquad (II.3-d)$$

(II.2) is an expression for the Fourier spectral components of the potential distribution on the surface of the substrate. Therefore the potential on the surface

$$\Phi(x,z)=(\frac{1}{2\pi})^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\overline{\phi}(k_x,k_z)e^{-jk_x x}e^{-jk_z z}dk_x dk_z+C \qquad (II.4-a)$$

can equivalently be written as

$$\Phi(x,z)=(\frac{1}{2\pi})^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\overline{G}(k_x,k_z)\cdot\overline{\rho}(k_x,k_z)e^{-ik_x x}e^{-jk_z z}+C \qquad (II.4-b)$$

$G(x,z)$ and $\overline{G}(k_x,k_z)$ are related by

$$G(x,z)=(\frac{1}{2\pi})^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\overline{G}(k_x,k_z)e^{-jk_x x}e^{-jk_z z}dk_x dk_z \qquad (II.5)$$

In (II.5) the integration must be interpreted in Cauchy's sense. (II.1), (II.4-b) and (II.5) are our basic equations for the calculation of the charge distribution on the metallic fingers on the surface. Following the same formalism as in the first part, we need an expression for $G(x,z)$. Using (II.5), changing to the polar-coordinate system and finally using the distributional equation, (II.6), [3]

$$\int_{0}^{+\infty}e^{-jk(xcos\phi+zsin\phi)}dk=\pi\frac{\delta(\phi-\phi_0)}{|-xsin\phi_0+zcos\phi_0|} \qquad (II.6)$$

we obtain

$$G(x,z)=\frac{\Gamma}{2\pi^2}\cdot\frac{1}{\sqrt{x^2+z^2+\sqrt{\alpha z^2-2\beta xz+\gamma x^2}}} \qquad (II.7)$$

In (II.6) ϕ_0 equals $arctan(-\frac{x}{z})$. A detailed discussion about $G(x,z)$ will be given elsewhere. Inserting the relation (II.7) in (II.1), approximating the charge distribution on the fingers by two-dimensional stepfunctions and interchanging the order of summation and integration and finally following the solution procedure discribed in part one, theoretically the problem is solved. Unfortunately the involved integrals are algebraically very complicated structured and could not be evaluated analytically, a fact which results in unacceptable computation times, if the method is applied to realistic SAW structures.

But what about Eq.(II.4-b)?

In fact if we use (II.4-b) instead of (II.1) we can show that the occuring integrals are easier to handle and can be calculated in closed-form.

III. Approximation of the charge density on the fingers

A nonequidistant discretization of the fingers which is appropriate to our specific problem is shown in Fig.2.

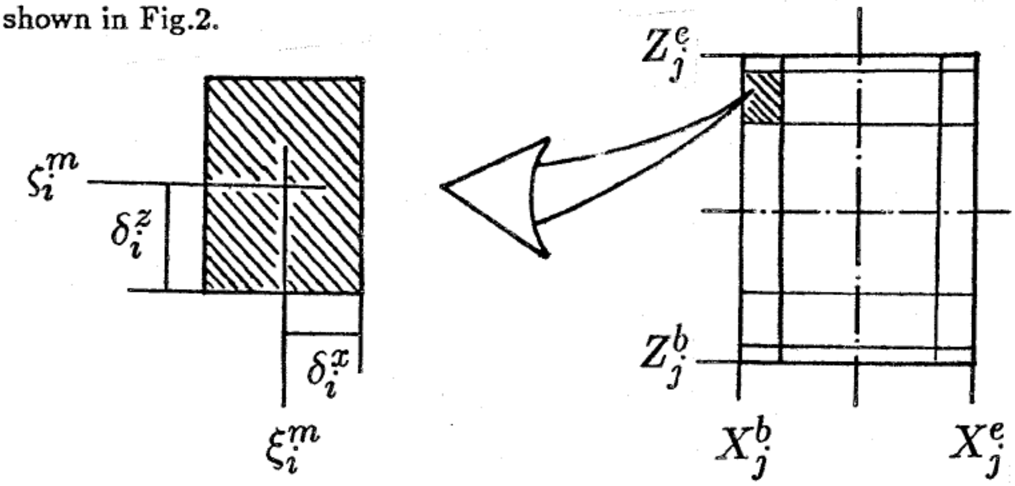

Fig.2: Discretization of j^{th} finger

For our calculations we have chosen a two dimensional stepfunction approximation for the charge density, which formally can be written as

$$\rho(x,z) = \rho_0 \sum_{j=1}^{N} \sigma_j f_j(x,z) \qquad (III.1-a)$$

with

$$f_j(x,z) = [H(x-x_j^b) - H(x-x_j^e)] \cdot [H(z-z_j^b) - H(z-z_j^e)] \qquad (III.1-a)$$

where

ρ_0 ... charge normalizing factor, N ... total number of finger subdivisions, σ_j ... charge on j^{th} subdivision H ... Heaviside's stepfunction

IV. Approximation of the potential distribution on the surface

Using the formula

$$\overline{\rho}(k_x,k_z) = (\frac{1}{2\pi})^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x,z)e^{jk_x x}e^{jk_z z}dk_x dk_z \qquad (IV.1-a)$$

and inserting $\rho(x,z)$, (III.1-a) in (IV.1-a) we obtain

$$\overline{\rho}(k_x,k_z) = \rho_0 \sum_{j=1}^{N} \frac{-e^{-ja}+e^{-jb}+e^{-jc}-e^{-jd}}{k_x k_z} \qquad (IV.1-b)$$

with

$$a = k_x \cdot (\xi_j^m + \Delta\xi_j) + k_z \cdot (\varsigma_j^m + \Delta\varsigma_j) \qquad (IV.1-c)$$

$$b = k_x \cdot (\xi_j^m + \Delta\xi_j) + k_z \cdot (\varsigma_j^m - \Delta\varsigma_j) \qquad (IV.1-d)$$

$$c = k_x \cdot (\xi_j^m - \Delta\xi_j) + k_z \cdot (\varsigma_j^m + \Delta\varsigma_j) \qquad (IV.1-e)$$

$$d = k_x \cdot (\xi_j^m - \Delta\xi_j) + k_z \cdot (\varsigma_j^m - \Delta\varsigma_j) \qquad (IV.1-d)$$

Insertion of $\overline{G}(k_x,k_z)$, (II.3-a) and $\overline{\rho}(k_x,k_z)$, (IV.1-b) in (II.4-b), changing the order of summation and integraion and changing to the polar coordinate system one can show that

$$\Phi(x,z) = \frac{\rho_0}{2\pi\epsilon_0}\sum_{j=1}^{N}\sigma_j\Delta\xi_j\Delta\varsigma_j \cdot A_j^*(x,z) + C \qquad (IV.2-a)$$

with

$$A_j^*(x,z) = \frac{1}{\Delta\xi_j\Delta\varsigma_j}\cdot(A_j^{(1)}(x,z)+A_j^{(2)}(x,z)+A_j^{(3)}(x,z)+A_j^{(4)}(x,z)) \qquad (IV.2-b)$$

$$A_j^{(1)}(x,z) = |a|\cdot I_T(\alpha,\beta,\gamma,-\frac{c}{a},-\frac{e}{a}) \qquad (IV.2-c)$$

$$A_j^{(2)}(x,z) = |b|\cdot I_T(\alpha,\beta,\gamma,-\frac{e}{b},-\frac{c}{b}) \qquad (IV.2-d)$$

$$A_j^{(3)}(x,z) = |c|\cdot I_T(\beta,\alpha,\gamma,-\frac{a}{c},-\frac{b}{c}) \qquad (IV.2-e)$$

$$A_j^{(4)}(x,z) = |d|\cdot I_T(\beta,\alpha,\gamma,-\frac{b}{d},-\frac{a}{d}) \qquad (IV.2-f)$$

$$a = x_j^b - x$$

$$b = x_j^e - x$$

$$c = z_j^b - z$$

$$d = z_j^e - z \qquad (IV.2-g)$$

and

$$I_T(p_1,p_2,p_3,u_1,u_2) = \int_{u_1}^{u_2}\frac{1}{\sqrt{p_1u^2+2p_2u+p_3}+\sqrt{u^2+1}}du \qquad (IV.2-h)$$

As briefly we have discussed in [1] the above integral can be calculated in closed form.

V. Point-Matching

According to the fact that the value of potential on the i^{th} subdivision is known and equals ϕ_i, ($\phi_i = \Phi(x = \xi_i^m, z = \varsigma_i^m)$), and using (IV.2-a) we obtain

$$\phi_i = \frac{\rho_0}{2\pi\epsilon_0}\sum_{j=1}^{N}\sigma_j\Delta\xi_j\Delta\varsigma_j \cdot A_j^*(\xi_i^m, \varsigma_i^m) + C; \quad i = 1 \ldots N \tag{V.1}$$

With $\frac{\rho_0}{2\pi\epsilon_0} = 1$ and $\sigma_j\Delta\xi_j\Delta\varsigma_j = q_j$ we get

$$\phi_i = \sum_{j=1}^{N} q_j \cdot A^*(i,j) + C; \quad i = 1, \ldots N \tag{V.2}$$

where we used $A^*(i,j)$ for $A_j^*(\xi_i^m, \varsigma_i^m)$.

VI. A Modified Inverse Capacitance Matrix

To explain the meaning of C In (VI.2-a) we introduce a reference transducer with the following properties:

i) the geometry of the fingers may show a quadrantal symmetry

ii) the potentials of the fingers must be quadrantal antisymmetric

One can show that for this reference transducer C is exactly zero. If a transducer considered relative to the above reference transducer has a structural or electrical mismatch, C is not equal zero. Therefore C can be regarded as a system mismatch parameter, which is apriori unknown.
Equating $C = q_{N+1}$ and $A^*(i, N+1) = 1$, (V.2) can be written as

$$\phi_i = \sum_{j=1}^{N+1} q_j \cdot A^*(i,j); \quad i = 1 \ldots N \tag{VI.1}$$

The explicit formulation of the charge neutrality condition,

$$\sum_{j=1}^{N} q_j = 0 \tag{VI.2}$$

together with (VI.1) these are $N+1$ equations for the $N+1$ unknowns q_j, ($j = 1 \ldots N+1$), which compactly can be written as

$$\underline{\phi}^* = (\underline{\underline{A}}^*)\underline{q}^* \tag{VI.3}$$

VII. Floating Fingers

In two dimensional representation the floating fingers are metallic plates, which are disconnected from the active fingers. Therefore their potentials are initially unknown and the charge integral on a floating finger equals zero.

Above we have regarded the system mismatch parameter C as a further component of the vector $\underline{q}^*$, which resulted to an additional column in $\underline{\underline{A}}^*$. Further to make the matrix equation invertable the charge neutrality condition was regarded as an additional row in $\underline{\underline{A}}^*$. The same procedure can be applied here. The unknown potential of a floating finger will be regarded as a further component of the $\underline{q}^*$ (corresponding to the addition of a column to $\underline{\underline{A}}^*$) and the charge neutrality condition of a floating finger can be taken into account as a further row in $\underline{\underline{A}}^*$, so we have

$$\underline{\phi} = (\underline{\underline{A}})\underline{q} \qquad (VII.1)$$

Results

Fig.3 and Fig.4 show two examples of our calculations based on the formalism described above.

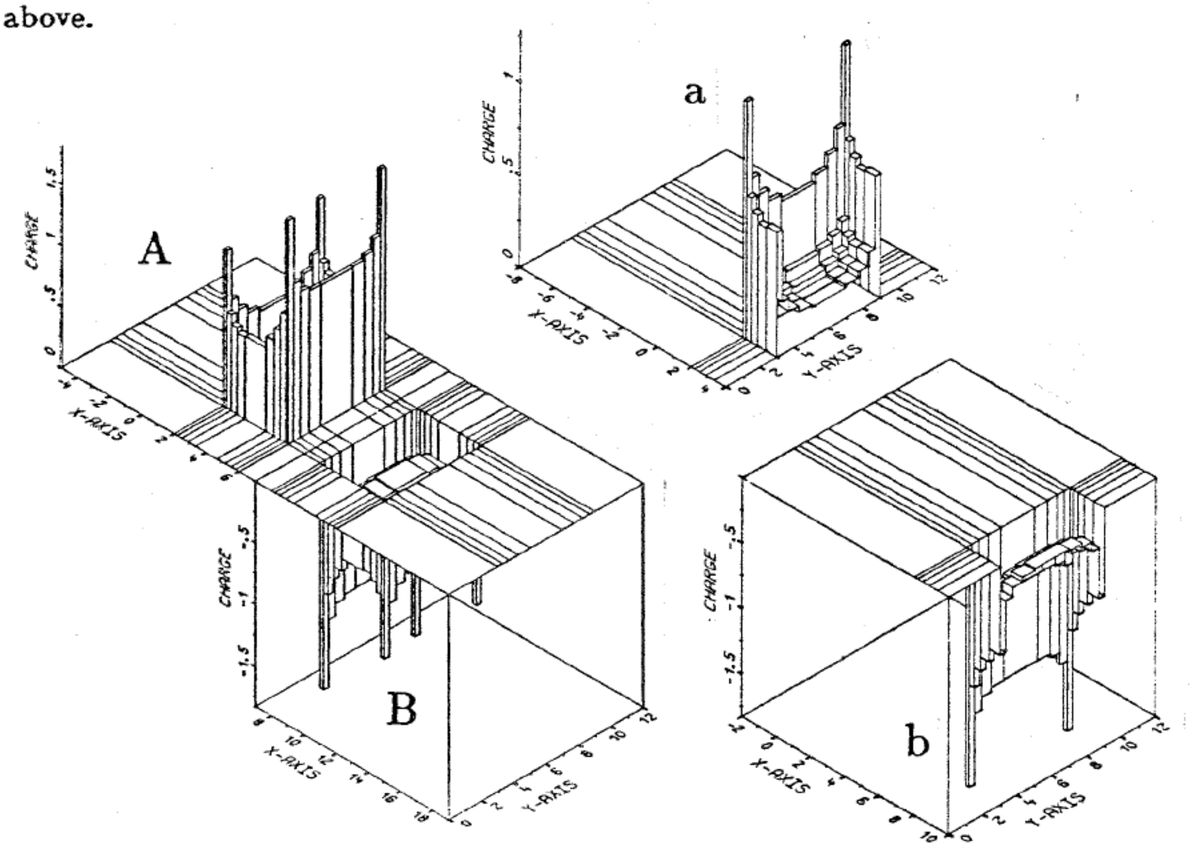

Fig.3 2D charge distribution on two parallel metallic plates on $LiNbO_3$ substrate.

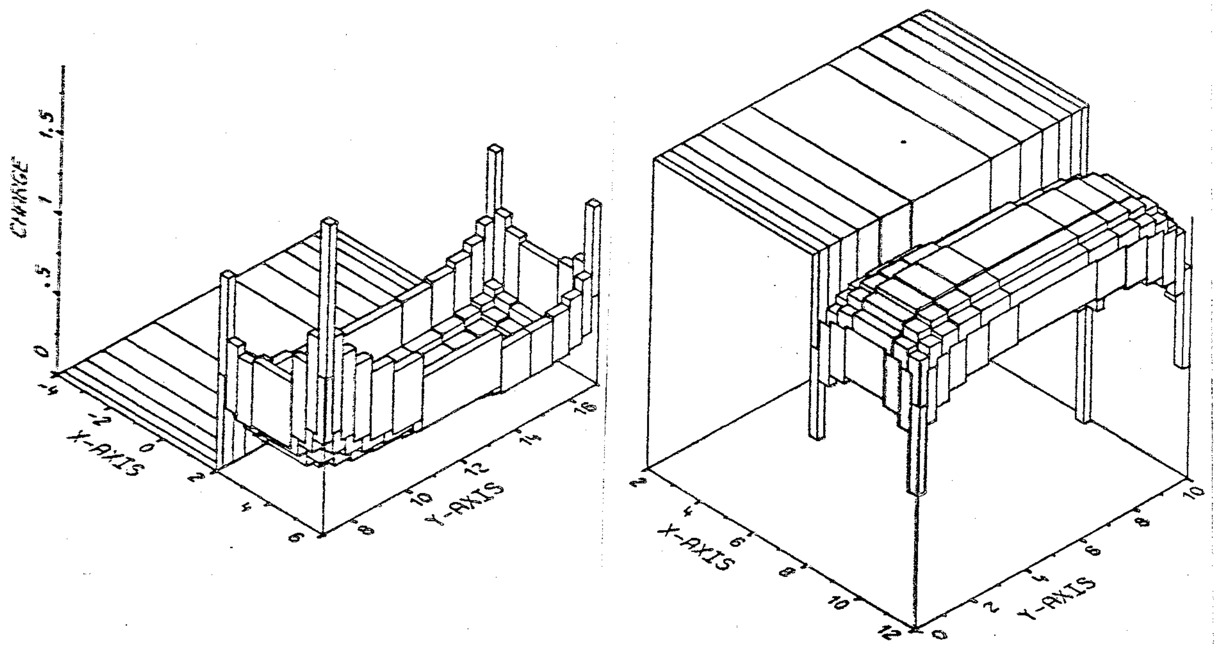

Fig.4 2D charge distribution on two parallel metallic plates on $LiNbO_3$ subtrate. The plates are shifted in transversal direction.

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