Electron Mobility Model for Strained-Si Devices

Siddhartha Dhar, Hans Kosina, *Member, IEEE*, Vassil Palankovski, Stephan Enzo Ungersboeck, and Siegfried Selberherr, *Fellow, IEEE*

***Abstract*—Strained-Si material has emerged as a strong contender for developing transistors for next-generation electronics, because this material system offers superior transport properties. We suggest a model describing the low-field bulk mobility tensor for electrons in strained-Si layers as a function of strain. Our analytical model includes the effect of strain-induced splitting of the conduction band valleys in Si, intervalley scattering, and doping dependence. Intervalley scattering has been modeled on the equilibrium electron distribution and the valley splitting for a given strain tensor. The effect of different substrate orientations is considered by performing coordinate transformations for the strain tensor and effective masses. Monte Carlo simulations accounting for various scattering mechanisms and the splitting of the anisotropic conduction band valleys due to strain in combination with an accurate ionized impurity scattering model were carried out to verify the results for the complete range of Ge contents and for a general orientation of the SiGe buffer layer. Our mobility model is suitable for implementation into a conventional technology CAD simulation tool.**

***Index Terms*—Intervalley scattering, mobility model, Monte Carlo simulations, SiGe, strained -Si, technology CAD.**

I. Introduction

IN THE last few decades, silicon has been the basis of manufacturing a majority of semiconductor devices. However, due to the continuous demand of higher integration densities, speed, and efficiency, device dimensions have been reduced to the extent where conventional Si approaches its fundamental limits. New materials exhibiting beneficial properties are being sought to overcome this problem. Much research in recent years has been driven toward strained-Si. Strain lifts the degeneracy of the valence and conduction bands which can be used to deliver superior transport properties in comparison to bulk Si. Transistors fabricated using strained-Si layers have reported larger drive currents capabilities [1]–[4] due to the enhanced electron and hole mobilities.

To enable the design of new device structures based on strained-Si, a reliable set of models for parameters such as mobility, energy bandgap, and relaxation times is required. This work suggests an analytical model to describe the {low-field bulk} mobility tensor for electrons in strained-Si layers as a function of strain. The model includes valley splitting for a given strain tensor, the effect of intervalley scattering, and the doping dependence. Monte Carlo simulations accounting for various scattering mechanisms and the splitting of anisotropic conduction bands were performed to verify the results for strained-Si for the complete range of germanium contents and for a general orientation of the SiGe substrate.

Manuscript received September 21, 2004; revised January 25, 2005. This work was supported by the Semiconductor Research Corporation (SRC) under Project 998.001. The review of this paper was arranged by Editor C. McAndrew.

S. Dhar, H. Kosina, S. E. Ungersböck, and S. Selberherr are with the Institut für Mikroelektronik, Technische Universität Wien, A-1040 Vienna, Austria (e-mail: dhar@iue.tuwien.ac.at).

V. Palankovski is with Infineon Technologies Austria, A-9500 Villach, Austria.

Digital Object Identifier 10.1109/TED.2005.844788

II. Theoretical Derivation

To develop an electron mobility model for silicon under different stress/strain conditions, the relative population of electrons in the different valleys in Si has to be properly considered [5], [6]. As suggested in [5], the anisotropic electron mobility in strained-SiGe–Si can be computed by taking the weighted average of the unstrained electron mobility tensor, $\hat{\mu}^{(i)}_{n,\mathrm{uns}}$ of the ith conduction band in Si with the corresponding electron concentration, $n^{(i)}_{\mathrm{str}}$ in the ith pair of valley in strained-Si,

$$\hat{\mu}^{\mathrm{tot}}_n = \sum_{i=1}^{3} p^{(i)} \cdot \hat{\mu}^{(i)}_{n,\mathrm{uns}}, \quad p^{(i)} = \frac{n^{(i)}_{\mathrm{str}}}{\sum_{i=1}^{3} n^{(i)}_{\mathrm{str}}} \tag{1}$$

$$n^{(i)}_{\mathrm{str}} = N^{(i)}_C \cdot \exp\left[\frac{\Delta E^{(i)}_C(y)}{k_B T}\right]. \tag{2}$$

Here, $n^{(i)}_{\mathrm{str}}$ is calculated for nondegenerate doping concentrations, using Boltzmann statistics with $N^{(i)}_C$ as the effective density of states and $\Delta E^{(i)}_C(y)$ as the energy shift for the ith valley, respectively. k_B and T denote the Boltzmann's constant and ambient temperature, respectively. The shifts in energies of the conduction band valleys is given by [7] as

$$\Delta E^{(i)}_C = \Xi_d(\varepsilon_{\mathrm{xx}} + \varepsilon_{\mathrm{yy}} + \varepsilon_{\mathrm{zz}}) + \Xi_u \varepsilon_{\mathrm{ii}}, \quad i = x, y, z \tag{3}$$

with Ξ_d and Ξ_u as the dilatation and shear deformation potentials for the conduction band and the $\varepsilon'_{\mathrm{ii}}s$ are the diagonal components of the strain tensor expressed in the principal coordinate system. Note that (3) holds for arbitrary stress/strain conditions, including uniaxial/biaxial strain.

Calculation of the in-plane and out-of-plane electron mobilities in strained-Si layer as a function of the Ge content y in the SiGe [001] substrates using the model in [5] is shown in Fig. 1. As can be seen from the figure, the model reproduces the linear increase (decrease) in the in-plane (out-of-plane) electron mobility component followed by mobility saturation for high strain levels, consistent with the theory. However, the model shows a relatively high value of the unstrained mobilities if the saturation mobility values are fixed. This is due to the fact that it does not consider the effect of intervalley scattering which is present in unstrained-Si and results in a lower mobility. The model in [5] uses only two parameters with which it is not possible to match the three quantities simultaneously such as the unstrained, in-plane, and perpendicular electron mobilities.

0018-9383/$20.00 © 2005 IEEE

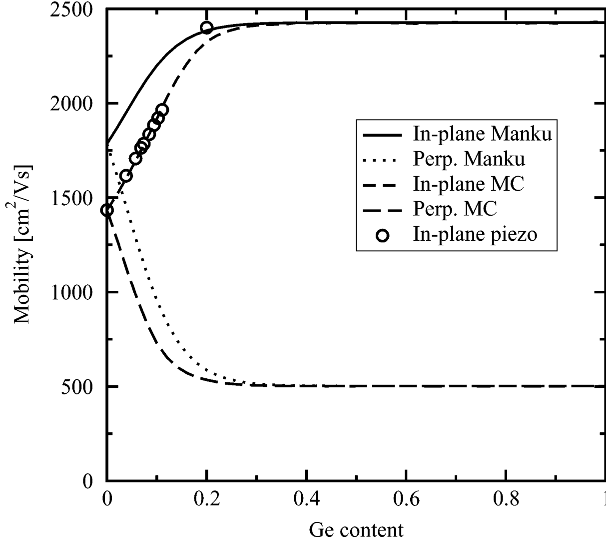

Fig. 1. In-plane and perpendicular electron mobilities in undoped strained-Si versus the Ge content in SiGe [001] buffer layer calculated using [5]. Also shown for comparison is the mobility obtained from piezo-resistance coefficients [9], [10].

To correctly model the electron mobility in strained-Si, including the effect of intervalley scattering, we modify (1) and write the total electron mobility as

$$\hat{\mu}_n^{\text{tot}} = \sum_{i=1}^{3} p^{(i)} \cdot \hat{\mu}_{n,\text{str}}^{(i)}. \tag{4}$$

Here, the $\hat{\mu}_{n,\text{str}}^{(i)}$ denotes the electron mobility tensors for strained-Si for [100], [010], and [001] X-valleys corresponding to directions x, y, and z, respectively. In (4) we model the mobility tensor as a product of a scalar mobility and the scaled inverse mass tensor

$$\hat{\mu}_{n,\text{str}}^{(i)} = \mu \cdot \hat{m}_{(i)}^{-1}, \quad i = x, y, z \tag{5}$$

$$\hat{m}_x^{-1} = \begin{pmatrix} \frac{m_c}{m_l} & 0 & 0 \\ 0 & \frac{m_c}{m_t} & 0 \\ 0 & 0 & \frac{m_c}{m_t} \end{pmatrix}, \quad \hat{m}_y^{-1} = \begin{pmatrix} \frac{m_c}{m_t} & 0 & 0 \\ 0 & \frac{m_c}{m_l} & 0 \\ 0 & 0 & \frac{m_c}{m_t} \end{pmatrix}$$
$$\hat{m}_z^{-1} = \begin{pmatrix} \frac{m_c}{m_t} & 0 & 0 \\ 0 & \frac{m_c}{m_t} & 0 \\ 0 & 0 & \frac{m_c}{m_l} \end{pmatrix}. \tag{6}$$

The factor μ includes the dependence on the energies $\Delta E_C^{(i)}$ and the doping concentration N_I in the strained-Si layer

$$\mu(N_I, \Delta E_C^{(i)}) = \frac{e}{m_c \left(\frac{1}{\tau_{\text{equiv}}} + \frac{1}{\tau_{\text{neq}}\left(\Delta E_C^{(i)}\right)} + \frac{1}{\tau_I(N_I)} \right)}. \tag{7}$$

In (7) τ_{equiv} denotes the momentum relaxation time due to acoustic intravalley scattering and intervalley scattering between equivalent valleys (g-type), $\tau_{\text{neq}}(\Delta E_C^{(i)})$ for intervalley scattering between nonequivalent valleys (f-type scattering), and $\tau_I(N_I)$ for impurity scattering. The effect of the different scattering mechanisms on the total mobility is estimated by Matthiesson's rule in (7). The tensors in (6) are the inverse effective mass tensors with m_t, m_l denoting the transversal and lateral masses for the ellipsoidal X-valleys in Si. The tensor is scaled to a dimensionless form by the conductivity mass m_c

$$m_c = \frac{3}{\frac{2}{m_t} + \frac{1}{m_l}}. \tag{8}$$

To arrive at a formal description of the mobility components in strained-Si in terms of measurable macroscopic quantities, the following cases are considered.

A. Electron Mobility in Unstrained and Undoped Si

The electron mobility for unstrained (relaxed) and undoped Si, also referred to as lattice mobility μ^L, can be derived by dropping the impurity scattering rate in (7)

$$\hat{\mu}_{n,\text{uns}} = \frac{e}{m_c \left(\frac{1}{\tau_{\text{equiv}}} + \frac{1}{\tau_{\text{neq}}^0} \right)} \sum_{i=1}^{3} \frac{1}{3} \cdot \hat{m}_{(i)}^{-1} = \mu^L \cdot \hat{I}. \tag{9}$$

Here, τ_{neq}^0 denotes the intervalley relaxation time for f type phonon scattering for electrons in unstrained-Si. Note that the sum evaluates to the identity matrix $\hat{I}$.

B. Electron Mobility in Strained and Undoped Si

When dealing with in-plane biaxial-tensile strain, the six-fold degenerate Δ_6-valleys in Si are split into two-fold degenerate Δ_2 valleys (lower in energy) and four-fold degenerate Δ_4 valleys (higher in energy) with electrons preferentially occupying the lower energy levels. As a result, the saturation values of the transversal (μ_t^{sat}) and lateral (μ_l^{sat}) mobilities of the ellipsoidal valleys in Si may be expressed as

$$\mu_t^{\text{sat}} = \frac{e \cdot \tau_{\text{equiv}}}{m_t}, \quad \mu_l^{\text{sat}} = \frac{e \cdot \tau_{\text{equiv}}}{m_l}. \tag{10}$$

Here, it is assumed that the strain-induced valley splitting (corresponding to $y \geq 0.4$ in the SiGe layer) is large enough such that the lowest valley is fully populated and the intervalley scattering to higher valleys is suppressed.

We introduce a mobility enhancement factor f which can be expressed as

$$f = \frac{\mu_t^{\text{sat}}}{\mu^L} = \frac{m_c}{m_t} \left(1 + \frac{\tau_{\text{equiv}}}{\tau_{\text{neq}}^0} \right). \tag{11}$$

C. Electron Mobility in Unstrained and Doped Si

In analogy with (9) the electron mobility for unstrained-Si with doping concentration N_I can be written as

$$\hat{\mu}^{LI} = \frac{e}{m_c \left(\frac{1}{\tau_{\text{equiv}}} + \frac{1}{\tau_{\text{neq}}^0} + \frac{1}{\tau_I(N_I)} \right)} \cdot \hat{I} = \mu^{LI} \cdot \hat{I} \tag{12}$$

where $\hat{\mu}^{LI}$ signifies the lattice mobility including the effect of impurity scattering. Using (11) and (12) we can write

$$\frac{\tau_{\text{equiv}}}{\tau_{N_I}} = \left(\frac{\mu^L}{\mu^{LI}} - 1 \right) \frac{m_t}{m_c} \cdot f. \tag{13}$$

TABLE I
DEFORMATION POTENTIAL Ξ_u. THE COUPLING CONSTANTS FOR INTER-VALLEY SCATTERING IN Si ARE IN [10^8 eV/cm]

Parameter	Values	Reference			
Ξ_u [eV]	8.86, 8.47, 9.2, 7.3, 10.5, 9.29	Ref [15]			
Type of scattering	Ref [12]	Ref [17]	Ref [13]	Ref [21]	This Work
g_1	0.5	0.5	-	19.2	0.454
g_2	0.8	0.8	4.0	-	0.727
g_3	11	11	8.0	-	10
f_1	0.3	0.3	2.5	19.2	0.36
f_2	2.0	2.0	-	-	2.4
f_3	2.0	2.0	8.0	-	2.4

D. Electron Mobility in Strained and Doped Si

The intervalley scattering rate is a function of the strain-induced splitting of the valleys and can be expressed by a dimensionless factor $h^{(i)}$.

$$\frac{1}{\tau_{\mathrm{neq}}^{(i)}} = \frac{h^{(i)}}{\tau_{\mathrm{neq}}^{0}}. \tag{14}$$

In strained-Si, the total intervalley scattering rate for electrons to scatter from initial valley i to final valleys j and l is given by

$$\frac{1}{\tau_{\mathrm{neq}}^{(i)}} = \frac{1}{\tau_{(i\to j)}} + \frac{1}{\tau_{(i\to l)}}. \tag{15}$$

For low electric fields, an equilibrium distribution function can be assumed and $\tau_{(i\to j)}$ can be calculated [8]

$$\frac{1}{\tau_{(i\to j)}} = \frac{\int_0^\infty S(\epsilon,\Delta_{ij})\cdot \exp\left(\frac{-\epsilon}{k_BT}\right) d\epsilon}{\int_0^\infty \exp\left(\frac{-\epsilon}{k_BT}\right) d\epsilon} \tag{16}$$

with the intervalley scattering rate S defined as

$$S(\epsilon,\Delta_{ij}) = C\cdot\left[\left(\epsilon-\Delta_{ij}^{\mathrm{emi}}\right)^{1/2} + \exp\left(\frac{\hbar\omega_{\mathrm{opt}}}{k_BT}\right) \times \left(\epsilon-\Delta_{ij}^{\mathrm{abs}}\right)^{1/2}\right] \tag{17}$$

$$\Delta_{ij}^{\mathrm{emi}} = \Delta E_C^{(j)} - \Delta E_C^{(i)} - \hbar\omega_{\mathrm{opt}} \tag{18}$$

$$\Delta_{ij}^{\mathrm{abs}} = \Delta E_C^{(j)} - \Delta E_C^{(i)} + \hbar\omega_{\mathrm{opt}}. \tag{19}$$

Here, $\hbar\omega_{\mathrm{opt}}$ denotes the phonon energy and $\Delta E_C^{(i)}$, the strain induced splitting, and C is a constant. Using these expressions, the factor $h^{(i)}$ in (14) is determined in (20), shown at the bottom of the page. The function g is defined as

$$g(z) = \begin{cases} \exp(-z)\cdot\Gamma\left(\frac{3}{2}\right) & \forall z>0 \\ \exp(-z)\cdot\Gamma\left(\frac{3}{2},-z\right) & \forall z<0 \end{cases}. \tag{21}$$

Here, $\Gamma(3/2) = (\sqrt{\pi}/2)$ and $\Gamma((3/2),-z)$ denotes the incomplete Gamma function.

Replacing the intervalley term in (7) by (14) and using (11) and (13), the electron mobility for the ith valley in strained Si can be written as

$$\hat{\mu}_{n,\mathrm{str}}^{(i)}\left(N_I,\Delta E_C^{(i)}\right) = \frac{\beta\cdot\mu^L}{1+(\beta-1)\cdot h^{(i)} + \beta\cdot\left(\frac{\mu^L}{\mu^{LI}}-1\right)}\cdot \hat{m}_{(i)}^{-1} \tag{22}$$

where $\hat{m}_{(i)}^{-1}$ denotes the scaled effective mass tensor for the ith valley in (6) and $\beta = (f\cdot m_t)/(m_c)$. Equation (22) is plugged into (4) to give the total mobility tensor for electrons in strained-Si as a function of doping concentration N_I and strain. The tensor in (22) is given in the principle coordinate system and has a diagonal form.

III. RESULTS AND DISCUSSION

In order to validate our model, Monte Carlo (MC) simulations were performed. The MC results obtained have been fit to experimental data which are available mainly in the form of piezo-resistance coefficients [9]. For low strain levels, the increase in the in-plane electron mobility is linear [10], characterized by the piezo-resistance coefficients. It was observed that changing the shear deformation potential Ξ_u from its original value of 9.29 eV [11], gave good agreement for low strain levels, because varying Ξ_u results in a variation of the conduction band valley shifts through (3) and hence of the electron concentration and mobility. Table I lists the reported values of the shear deformation potential from which a value of 7.3 eV was chosen.

The existing transport models exhibit a considerable quantitative difference in the maximum increase of the in-plane electron mobility in biaxially strained Si layers when compared to conventional Si. Enhancement values ranging from 56% [12]

$$h^{(i)} = \frac{g\left(\frac{\Delta_{ij}^{\mathrm{emi}}}{k_BT}\right) + g\left(\frac{\Delta_{il}^{\mathrm{emi}}}{k_BT}\right) + \exp\left(\frac{\hbar\omega_{\mathrm{opt}}}{k_BT}\right)\left[g\left(\frac{\Delta_{ij}^{\mathrm{abs}}}{k_BT}\right) + g\left(\frac{\Delta_{ij}^{\mathrm{abs}}}{k_BT}\right)\right]}{2\left[g\left(\frac{-\hbar\omega_{\mathrm{opt}}}{k_BT}\right) + \Gamma\left(\frac{3}{2}\right)\right]} \tag{20}$$

up to 180% [13] have been simulated while measurements indicate a value of around 97% [14]. We have adopted a somewhat conservative value of 70% for the enhancement factor, in-between other reported values [15], [16]. To achieve such values, it was required to adjust the coupling constants of the g-type and f-type phonons [17]. The reported values of the coupling constants for intervalley scattering in Si are shown in Table I. Since at saturation values of electron mobilities, the f-type phonon scattering is completely suppressed, the only way to obtain an increase in the mobility is by reducing the g-type coupling constants. This adjustment however would result in increased value of the unstrained electron mobility. Therefore, it was required to increase the f-type phonon coupling constant to restore the unstrained mobility value. The final result was that the coupling constants of the g-type phonons were reduced by a factor of 1.1 and that for f-type phonons increased by a factor of 1.2, as compared to the original values proposed by Jacoboni [17]. The only parameter of the intervalley scattering model (22) is the phonon energy. A value of $\hbar\omega_{\rm opt} = 60$ meV has been assumed. While the model based on piezo-resistance coefficients assumes a linear relationship between mobility enhancement and strain, with the proposed model the mobility enhancement saturates at large strain values, as is observed experimentally.

For any arbitrary orientation of the SiGe substrate, one can assign a unit vector, $\vec{n}$, normal to the interface and two unit vectors, $\vec{a}$ and $\vec{b}$, defining the interface plane. Using these three column vectors the transformation matrix $\hat{U}(\alpha,\beta,\gamma)$ in [18] may be rewritten as

$$\hat{U}(\alpha,\beta,\gamma) = (\vec{a}, \quad \vec{b}, \quad \vec{n}) \tag{23}$$

with $\vec{a}, \vec{b}$, and $\vec{n}$ given by

$$\vec{a} = \begin{pmatrix} \cos(\alpha)\cos(\beta)\cos(\gamma) - \sin(\alpha)\sin(\gamma) \\ \sin(\alpha)\cos(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma) \\ -\sin(\beta)\cos(\gamma) \end{pmatrix} \tag{24}$$

$$\vec{b} = \begin{pmatrix} -\cos(\alpha)\cos(\beta)\sin(\gamma) - \sin(\alpha)\cos(\gamma) \\ -\sin(\alpha)\cos(\beta)\sin(\gamma) + \cos(\alpha)\cos(\gamma) \\ \sin(\beta)\sin(\gamma) \end{pmatrix}$$

$$\vec{n} = \begin{pmatrix} \cos(\alpha)\sin(\beta) \\ \sin(\alpha)\sin(\beta) \\ \cos(\beta) \end{pmatrix}. \tag{25}$$

Here α, β, and γ denote the three Euler angles obtained by the clockwise rotation about the axes as described in [18] with γ being the in-plane angle.

The dependence of the electron mobility components on the orientation of the underlying SiGe layer is taken into account by performing a transformation of the strain tensor from the interface coordinate system to the principle coordinate system, using the transformation matrix (23). Figs. 2–4 show the electron lattice mobility components for different strain levels obtained using (22) for substrate orientations [001], [110], and [123], respectively. As can be seen in Fig. 2, for substrate orientation [001] the two in-plane components of electron mobility are equal with the maximum mobility saturating to a value above 2400 $\mathrm{cm^2/Vs}$ at about 30% Ge content. For [110] orientation (see Fig. 3), the MC simulation results demonstrate that one of the in-plane components of the electron mobility is equal to the

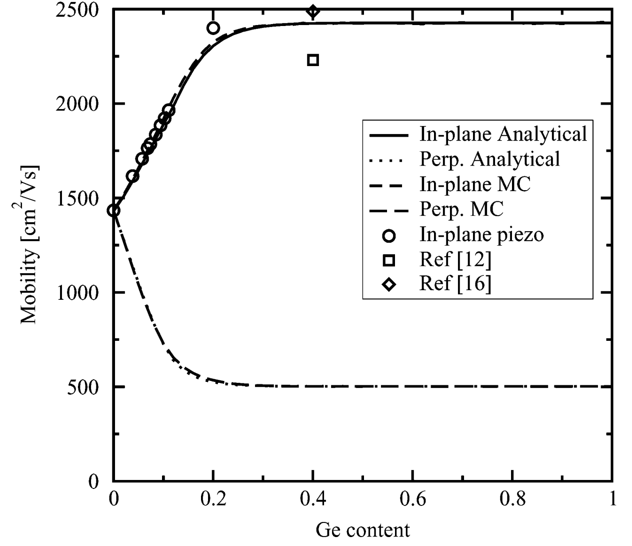

Fig. 2. In-plane and perpendicular electron mobilities in undoped strained-Si versus the Ge content in SiGe [001] buffer layer calculated using (22) together with the mobility obtained from piezo-resistance coefficients.

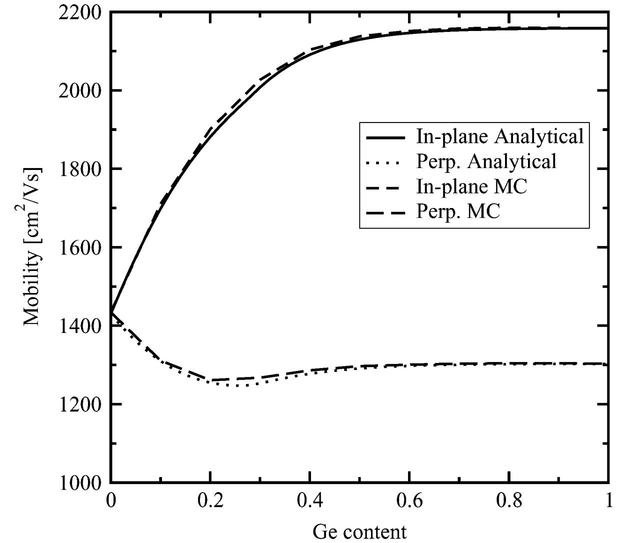

Fig. 3. In-plane and perpendicular electron mobilities in undoped strained-Si versus the Ge content in SiGe [110] buffer layer calculated using (22).

perpendicular component. This feature is also reproduced by the analytical model. The three distinct components of the mobility can be clearly seen for the [123] orientation (see Fig. 4) and our model shows excellent agreement with the MC simulation results for a large range of Ge content y in the SiGe layer}. The deviation for very large strain levels (Ge content ≥ 0.85) is due to the fact that the proposed model does not consider the population of the L valleys for large Ge content.

The dependence of the in-plane electron mobility in strained-Si on the in-plane angle γ can be obtained by taking the projection of the mobility tensor, $\hat{\mu}_n^{\rm tot}(N_I, y)$ in the direction of one of the in-plane vectors, $\vec{a}$ or $\vec{b}$ as

$$\mu(\gamma) = \vec{a}^T(\gamma) \cdot \hat{\mu}_n^{\rm tot}(N_I, y) \cdot \vec{a}(\gamma). \tag{26}$$

Figs. 5 and 6 show the variation of the mobility as a function of the angle γ for orientations [110] and [123] of SiGe substrate, calculated using (26). The in-plane vectors, $\vec{u}$ and $\vec{v}$ at which the

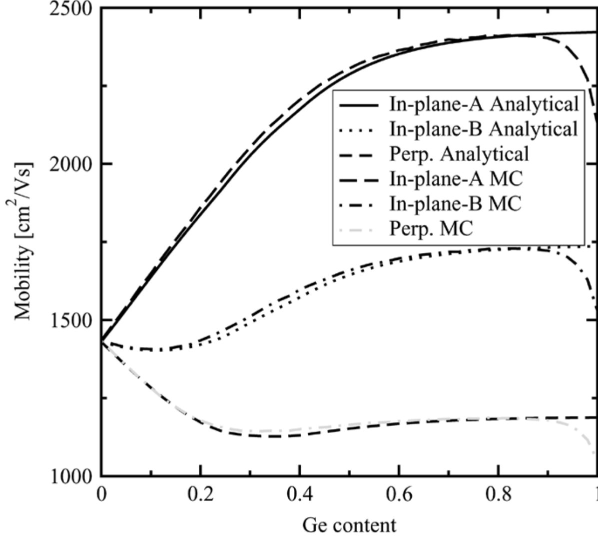

Fig. 4. In-plane and perpendicular electron mobilities in undoped strained-Si versus the Ge content in SiGe [123] buffer layer calculated using (22).

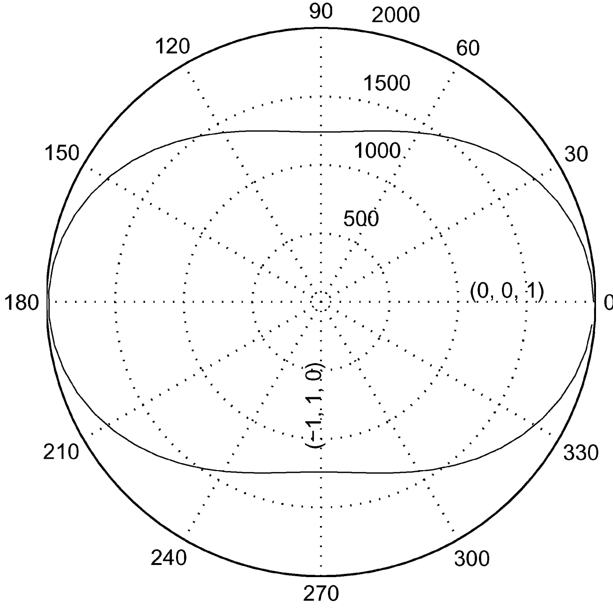

Fig. 5. In-plane electron mobilities (in cm²/Vs) in undoped strained-Si versus γ for SiGe [110] buffer layer ($y = 0.3$) calculated using (26).

electron mobility has its maximum and minimum values can be obtained by setting the derivative of (26) to zero. Alternatively one may choose any two orthogonal in-plane vectors $\vec{m}$ and $\vec{l}$ and calculate the projection of the mobility tensor

$$\mu_{ml} = \vec{m}^T \cdot \hat{\mu}_n^{\rm tot}(N_I, y) \cdot \vec{l}. \tag{27}$$

The vectors $\vec{u}$ and $\vec{v}$ can then be obtained by calculating the eigenvalues and eigenvectors for the following matrix:

$$\begin{pmatrix} \mu_{mm} & \mu_{ml} \\ \mu_{lm} & \mu_{ll} \end{pmatrix}. \tag{28}$$

The doping and material composition dependence of the in-plane and perpendicular electron mobilities in strained-Si is

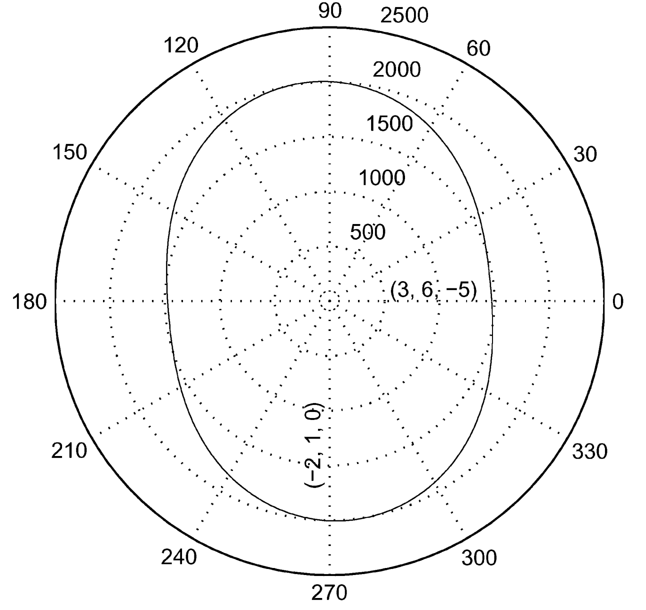

Fig. 6. In-plane electron mobilities (in cm²/Vs) in undoped strained-Si versus γ for SiGe [123] buffer layer ($y = 0.3$) calculated using (26).

TABLE II
PARAMETER VALUES FOR THE ELECTRON MOBILITY IN Si AT 300 K

Parameter	Si
$\mu_n^{\rm L}$ [cm²/Vs]	1430
$\mu_{\rm maj}^{\rm mid}$ [cm²/Vs]	44
$\mu_{\rm maj}^{\rm hi}$ [cm²/Vs]	57
$\mu_{\rm min}^{\rm mid}$ [cm²/Vs]	141
$\mu_{\rm min}^{\rm hi}$ [cm²/Vs]	218
η	0.65
λ	2.0
$C^{\rm mid}$ [cm^{-3}]	1.12e17
$C_{\rm maj}^{\rm hi}$ [cm^{-3}]	1.18e20
$C_{\rm min}^{\rm hi}$ [cm^{-3}]	4.35e19

calculated using (22) with the doping dependence of $\mu^{\rm LI}$ for minority and majority electrons in Si given by [19]

$$\mu_{n,\rm min}^{\rm LI} = \frac{\mu_n^{\rm L} - \mu_{\rm min}^{\rm mid}}{1 + \left(\frac{N_{\rm A}}{C^{\rm mid}}\right)^{\eta}} + \frac{\mu_{\rm min}^{\rm mid} - \mu_{\rm min}^{\rm hi}}{1 + \left(\frac{N_{\rm A}}{C_{\rm min}^{\rm hi}}\right)^{\lambda}} + \mu_{\rm min}^{\rm hi} \tag{29}$$

$$\mu_{n,\rm maj}^{\rm LI} = \frac{\mu_n^{\rm L} - \mu_{\rm maj}^{\rm mid}}{1 + \left(\frac{N_{\rm A}}{C^{\rm maj}}\right)^{\eta}} + \frac{\mu_{\rm maj}^{\rm mid} - \mu_{\rm maj}^{\rm hi}}{1 + \left(\frac{N_{\rm A}}{C_{\rm maj}^{\rm hi}}\right)^{\lambda}} + \mu_{\rm maj}^{\rm hi} \tag{30}$$

where μ_n^L is the mobility for undoped material and $\mu^{\rm hi}$ is the mobility at highest doping, $\mu_{\rm maj}^{\rm mid}, \mu_{\rm maj}^{\rm hi}, \mu_{\rm min}^{\rm mid}, \mu_{\rm min}^{\rm hi}, C^{\rm mid}, C_{\rm maj}^{\rm hi}, C_{\rm min}^{\rm hi}, \eta$, and λ are used as fitting parameters. The difference between majority and minority electron mobilities [20] is a well-known phenomenon caused by effects such as degeneracy and the different screening behavior of electrons and holes in the semiconductor. The model parameters are summarized in Table II.

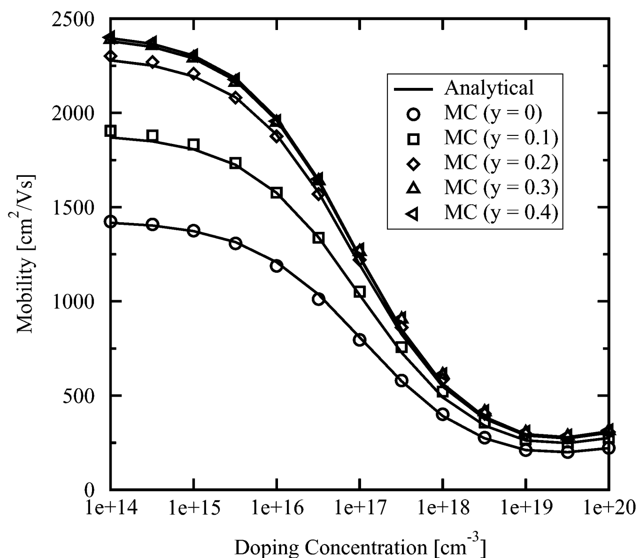

Fig. 7. Doping dependence of In-plane electron (minority) mobility in strained-Si calculated using (22) for different Ge content in SiGe [001] substrate.

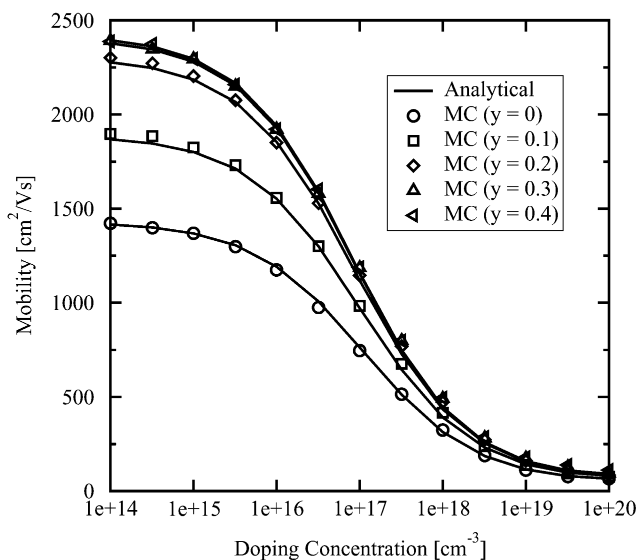

Fig. 8. Doping dependence of In-plane electron (majority) mobility in strained-Si calculated using (22) for different Ge content in SiGe [001] substrate.

Figs. 7 and 8 show the doping dependence of the in-plane minority and majority electron mobility components in strained-Si layers for different Ge content in the underlying SiGe for [001] orientation of the substrate. The solid lines depict the results as obtained from the analytical model (22), while the symbols indicate the MC simulation results. As can be seen, the model reproduces the increase in minority electron mobility for high doping concentrations for all strain levels, when compared to majority electron mobility.

IV. Conclusion

An analytical model has been developed to describe the anisotropy of the electron mobility in strained-Si. The model includes the doping dependence and the effect of reduction of intervalley scattering due to valley splitting. Results obtained from the model show excellent agreement with MC simulations for different SiGe substrate orientations and doping concentrations in the strained layer.

References

[1] L.-J. Huang, J. Chu, S. Goma, C. Emic, S. Koester, D. Canaperi, P. Mooney, S. Cordes, J. Speidell, R. Anderson, and H. Wong, "Carrier mobility enhancement in strained-Si-on-insulator fabricated by wafer bonding," in *VLSI Symp. Tech. Dig.*, 2001, pp. 57–58.

[2] K. Rim, J. Chu, H. Chen, K. Jenkins, T. Kanarsky, K. Lee, A. Mocuta, H. Zhu, R. Roy, J. Newbury, J. Ott, K. Petrarca, P. Mooney, D. Lacey, S. Koester, K. Chan, D. Boyd, M. Ieong, and H. Wong, "Characteristics and device design of sub-100 nm strained-Si N- and PMOSFETs," in *VLSI Symp. Tech. Dig.*, 2002, pp. 98–99.

[3] N. Sugii, D. Hisamoto, K. Washio, N. Yokoyama, and S. Kimura, "Enhanced performance of strained-Si MOSFETs on CMP SiGe virtual substrate," in *IEDM Tech. Dig.*, 2001, pp. 737–740.

[4] T. Mizuno, S. Takagi, N. Sugiyama, H. Satake, A. Kurobe, and A. Triumi, "Electron and hole mobility enhancement in strained-Si MOSFET's on SiGe-on-insulator substrates fabricated by SIMOX technology," *IEEE Electron Device Lett.*, vol. 21, no. 5, pp. 230–232, May 2000.

[5] T. Manku and A. Nathan, "Electon drift mobility model for devices based on unstrained and coherently strained-$Si_{1-x}Ge_x$ grown on (001) silicon substrates," *IEEE Trans. Electron Devices*, vol. 39, no. 9, pp. 2082–2089, Sep. 1992.

[6] J. Egley and D. Chidambarao, "Strain effects on devide characteristics: Implementation in drift-difusion simulators," *Solid State Electron.*, vol. 36, no. 12, pp. 1653–1664, 1993.

[7] I. Balslev, "Influence of uniaxial stress on the indirect absorption edge in silicon and germanium," *Phys. Rev.*, vol. 143, pp. 636–647, 1966.

[8] E. Conwel, *High Field Transport in Semiconductors*. New York: Academic, 1967.

[9] C. Smith, "Piezoresistance effect in germanium and silicon," *Phys. Rev.*, vol. 94, pp. 42–49, 1954.

[10] Y. Kanda, "A graphical representation of the piezoresistance coefficients in silicon," *IEEE Trans. Electron Devices*, vol. 29, no. 1, pp. 64–70, Jan. 1982.

[11] M. Rieger and P. Vogl, "Electronic-band parameters in strained-$Si_{1-x}Ge_x$ alloys on $Si_{1-y}Ge_y$ substrates," *Phys. Rev. B*, vol. 48, no. 19, pp. 14 276–14 287, 1993.

[12] F. Bufler, P. Graf, S. Keith, and B. Meinerzhagen, "Full band Monte Carlo investigation of electron transport in strained-Si grown on $Si_{1-x}Ge_x$ substrates," *Appl. Phys. Lett.*, vol. 70, pp. 2144–2146, 1997.

[13] T. Yamada, J. Zhou, H. Miyata, and D. Ferry, "In-plane transport properties of $Si/Si_{1-x}Ge_x$ structure and its FET performance by computer simulation," *IEEE Trans. Electron Devices*, vol. 41, no. 9, pp. 1513–1522, Sep. 1994.

[14] K. Ismail, S. Nelson, J. Chu, and B. Meyerson, "Electron transport properties of Si/SiGe heterostructures: Measurements and device applications," *Appl. Phys. Lett.*, vol. 63, pp. 660–662, 1993.

[15] M. Fischetti and S. Laux, "Band structure, deformation potentials, and carrier mobility in strained-Si, Ge, and SiGe alloys," *J. Appl. Phys.*, vol. 80, no. 4, pp. 2234–2252, 1996.

[16] T. Vogelsang and K. Hofmann, "Electron transport in strained-Si layers on $Si_{1-x}Ge_x$ substrates," *Appl. Phys. Lett.*, vol. 63, no. 2, pp. 186–188, 1993.

[17] C. Jacoboni and L. Reggiani, "The Monte Carlo method for the solution of charge transport in semiconductors with applications to covalent materials," *Rev. Mod. Phys.*, vol. 55, pp. 645–705, 1983.

[18] S. Smirnov and H. Kosina, "Monte Carlo modeling of the electron mobility in strained-$Si_{1-x}Ge_x$ layers on arbitrarily oriented $Si_{1-y}Ge_y$ substrates," *Solid State Electron.*, vol. 48, no. 1–4, pp. 1325–1335, 2004.

[19] V. Palankovski and R. Quay, *Analysis and Simulation of Heterostructure Devices*. Berlin, Germany: Springer-Verlag, 2004.

[20] G. Masetti, M. Severi, and S. Solmi, "Modeling of carrier mobility against carrier concentration in arsenic-, phosphorus-, and boron-doped silicon," *IEEE Trans. Electron Devices*, vol. ED-30, no. 7, pp. 764–769, Jul. 1983.

[21] S. Takagi, J. Hoyt, J. Welser, and J. Gibbons, "Comparative study of phononlimited mobility of two-dimensional electrons in strained and unstrained-Si metal-oxide-semiconductor field-effect transistors," *J. Appl. Phys.*, vol. 80, no. 3, pp. 1567–77, 1996.

Siddhartha Dhar was born in New Delhi, India, in 1979. He received the B.E. degree in electrical engineering from the Delhi College of Engineering in 2001, and the M.Sc. degree in microelectronics and microsystems from the Technical University of Hamburg, Hamburg, Germany, in 2003. He is currently pursuing the Ph.D. degree at the Institut für Mikroelektronik, Technische Universität Wien, Vienna, Austria.

His research interests include device modeling and simulation of strained-Si CMOS transistors and circuit level simulation in general.

Hans Kosina (M'92) was born in Haidershofen, Austria, in 1961. He received the Diplomingenieur degree in electrical engineering and the Ph.D. degree from the Institut für Mikroelektronik, Technische Universität Wien, Vienna, Austria, in 1987 and 1992, respectively.

For one year, he was with the Institut für Flexible Automation, and since 1988, he has been with the Institut für Mikroelektronik. In summer 1993, he held a Visiting Research position at the Advanced Products Research and Development Laboratory, Motorola, Austin, TX, and in summer 1999, held a Research position at the TCAD Department, Intel, Santa Clara, CA. His current interests include device simulation, modeling of carrier transport and quantum effects in semiconductor devices, new Monte Carlo algorithms, and computer-aided engineering in VLSI technology.

Dr. Kosina received the "Venia Docendi" in the field of microelectronics in 1998.

Vassil Palankovski was born in Sofia, Bulgaria, in 1969. He received the diploma degree in electrical engineering from the Technical University of Sofia in 1993, and the Ph.D. degree in technical sciences from the Institut für Mikroelektronik, Technische Universität Wien, Vienna, Austria, in 2000.

In 1997, after having worked for three years in the telecommunications field, he is currently with the Institut für Mikroelektronik as a Postdoctoral Researcher. In summer 2000, he held a Visiting Research position at LSI Logic Corp., Milpitas, CA. In autumn 2004, he was with Infineon Technologies AG, Villach, Austria. His scientific interests include device and circuit simulation, heterostructure device modeling, and physical aspects in general.

Stephan Enzo Ungersboeck was born in Vienna, Austria, in 1977. He received the Diplomingenieur degree in physics from Institut für Mikroelektronik, Technische Universität Wien, Vienna, in May 2002, where he is currently pursuing the Ph.D. degree.

He held a Visiting Research position at the Samsung Advanced Institute of Technology, Seoul, Korea, in summer 2003. His scientific interests include Monte Carlo simulation, band structure calculations, simulation of carbon nanotubes, and quantum mechanical confinement in submicrometer MOSFETs.

Siegfried Selberherr (M'79–SM'84–F'93) was born in Klosterneuburg, Austria, in 1955. He received the Diplomingenieur degree in electrical engineering and the Ph.D. degree in technical sciences from the Institut für Mikroelektronik, Technische Universität Wien, Vienna, Austria, in 1978 and 1981, respectively.

Since 1988, he has been the Head of the Institut für Mikroelektronik and since 1999, he has been Dean of the Fakultät für Elektrotechnik und Informationstechnik, Technische Universität Wien. His current research interests are modeling and simulation of problems for microelectronics engineering.

Dr. Selberherr has been holding the "Venia Docendi" on computer-aided design since 1984.