Shot Noise Suppression and Enhancement at 2D Hopping and in Single-Electron Arrays

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Abstract. We have carried out numerical simulations of shot noise at two-dimensional (2D) hopping and in 2D arrays of single-electron islands with and without random background charges. Such key transport characteristics as the average (dc) current $\langle I\rangle$, the single-particle density of states and the current fluctuation spectrum, have been calculated within a broad range of the applied electric field E and temperature T. Substantial Coulomb interaction effects are shown to not only suppress the average value of hopping current, but also affect its fluctuations rather substantially. In particular, at sufficiently low frequencies ($f \to 0$) the spectral density $S_I(f)$ of current fluctuations exhibits a $1/f$-like increase which approximately follows the Hooge scaling. As f increases, there is a crossover to a broad range of frequencies in which $S_I(f)$ is nearly constant, hence allowing characterization of the current noise by the Fano factor $F \equiv S_I(f)/2e\langle I\rangle$. For sufficiently large samples and low temperature, the Fano factor is suppressed below the Schottky value ($F = 1$), scaling with the sample length L as $F = (L_c/L)^{\alpha}$. The exponent α is significantly affected by the inclusion of Coulomb interaction effects, changing from $\alpha = 0.76 \pm 0.08$ when such effects are negligible to virtually unity when they are substantial. The scaling parameter L_c, interpreted as the average percolation cluster length along the electric field direction, scales as $L_c \propto E^{-(0.98\pm0.08)}$ when Coulomb interaction effects are negligible and $L_c \propto E^{-(1.26\pm0.15)}$ when such effects are substantial, in good agreement with results of directed percolation theory.

In arrays of single-electron islands with completely random background charges, the current noise is strongly colored at low currents, and its spectral density levels off at very low frequencies. The Fano factor may be much larger than unity, due to the remnants of single-electron/hole avalanches. However, even very small thermal fluctuations reduced the Fano factor below unity for almost any bias.

Keywords: Shot noise suppression, Coulomb blockade, single-electron arrays, Coulomb gap, hopping
PACS: 72.20.Ee, 72.20.Ht, 72.70.+m, 73.23.Hk, 73.40.Rw, 85.35.Gv

INTRODUCTION

Due to the fast approach of conventional field effect transistors (FET's) to their scaling limits, single-electron transistors (SET's) and arrays of tunnel junctions have begun attracting significant attention as possible FET alternatives. In fact, the arrays are of particular interest, since they may not only be used to replace the double junctions in SET's, hence raising the single-electron energy and operating temperature, but due to their high resistance may potentially be used as a device to leak parasitic random background charge, usually a fraction of the electron charge e, away from SET single-

CP800, *Unsolved Problems of Noise and Fluctuations: UPoN 2005*
edited by L. Regiani, C. Ponnetta, V. Akimov, E. Alfinito, and M. Rosini
2005 American Insitute of Physics 0-7354-0289-2/05/$22.50

electron islands. It is the latter capability which is the motivating factor behind this work.

For a device to dissolve such fractional random background charge, the transport through it must be quasi-continuous. One requirement of such quasi-continuous charge transfer was formulated in Ref. [1] in the perhaps most interesting limit of negligible thermal and quantum fluctuations and uses the fact that the dynamical fluctuations of the current flowing through a mesoscopic system are more sensitive to the charge transport mechanism peculiarities than the average transport characteristics, therefore providing additional information about the conduction physics [2, 3]. In particular, the low temperature broadband current fluctuations ("shot noise") are caused by the discreteness of the charge carriers and may be characterized using the spectral density $S_I(f)$ of current I fluctuations at low frequency f. In order to transport charge quasi-continuously [1] the shot noise must be suppressed. The Fano factor definition $F \equiv S_I(0)/2eI$ is found to be convenient, since it must be considerably smaller than the discrete electron transfer (Schottky) value $F = 1$.

An alternative to a 2D array as a prospective device candidate is a 2D hopping conductor, which is potentially more promising in applications than the array due to its very high resistance. Although the variable range hopping mechanism had been described a few decades ago [4, 5, 6], little was known about its current noise properties due to the complexity of charge flow in a completely disordered system. Even now due to the needs of large-scale statistical averaging with respect to a broad range of random parameters, such as site positions, on-site energies and random background charge, supercomputers are required to simulate the charge transfer process and calculate the current noise characteristics of interest.

We present our results of extensive numerical simulations of shot noise in 2D arrays of single-electron islands and at 2D hopping within a broad range of electric fields E and temperatures T.

SHOT NOISE ENHANCEMENT IN SINGLE-ELECTRON ARRAYS

Arrays of small conducting islands separated by tunnel junctions are among the simplest systems exhibiting Coulomb blockade and the related effects of correlated single-electron tunneling.

Figure 1 shows the Fano factor F as a function of dc current I per channel for several arrays of length $N = 20$ and width $M = 20$ islands, with completely random distributions of background charges. As dc current is reduced and/or the array size is increased, the frequency at which $S_I(\omega)$ levels off decreases rather dramatically, while the saturation level $S_I(0)$ increases well beyond the Schottky value $2eI$. In 1D arrays (see inset of Fig. 1), $S_I(0)$ stays always below $2eI$, so that the Fano factor F is always below unity ($F < 1$) [1]. For $T = 0$ and $I \to 0$, the Fano factor saturates for any particular system as $f \to 0$, but at a very high level, with exponentially broad statistical distribution. At large currents the results are virtually independent of the system dimensionality; in fact, general arguments [1, 7] show that in all cases $S_I(\omega)$ tends to $2eI/N$.

The difference between results for 2D and 1D arrays may be attributed to the different topology of charge carrier motion. In the 1D case, current is carried along the array by a single stream of individual electrons (holes). The array provides a bottleneck for this

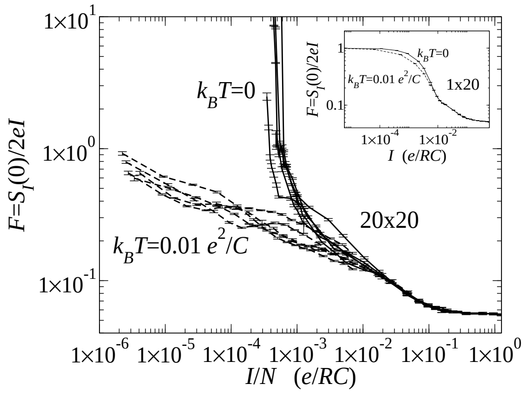

FIGURE 1. Fano factor F as a function of dc current I per channel for several 20×20 arrays, at zero and finite temperatures. The inset shows the corresponding dependences in a 1D array.

motion. At $I \to 0$, overcoming this bottleneck becomes a dominating source of current fluctuations, so that the current fluctuations are essentially the same as in a single tunnel junction, i.e. of shot noise character with $F \to 1$ [1].

On the contrary, broad 2D arrays may feature multiple streams of electrons and holes moving in opposite directions, that do not necessarily annihilate even in the absence of disorder, giving rise to long "avalanches" producing very large noise ($F \gg 1$) [8]. The disorder suppresses this effect (Fig. 1), but only partly.

It is important for applications that the Fano factor at low currents is substantially suppressed by even very small thermal fluctuations. For example, Fig. 1 shows that the temperature as low as $k_BT = 10^{-2}e^2/C$ reduces the Fano factor at low currents by several orders of magnitude, bringing it below unity, except for a very small region where $eV < 2k_BT$, where the current fluctuations are essentially thermal, rather than due to the shot noise. This behavior is very much different from that of 1D arrays (inset) where a small increase of temperature has a negligible effect on the shot noise.

NOISE AT HOPPING

Our simulations of hopping transport through 2D conductors of different geometries for a moderately strong Coulomb interaction strength $\chi = (\nu_0 a e^2/\kappa) = 0.5$ (with a the localization radius, ν_0 the density of states and κ the dielectric constant) has allowed us to calculate the frequency dependence of the current spectral density $S_I(\omega)$ (Fig. 2).

Two frequency ranges must be distinguished. At very low frequencies, the sharp $1/f$-type increase in noise is seen. This $1/f$-type contribution has been observed experimentally in a wide variety of conductors [3] and is observed to be proportional to the square of the current and in many cases found to scale approximately in accordance with the phenomenological Hooge formula [3, 9], which in 2D can be presented as

$$S_I(f)/\langle I\rangle^2 = \left(a^2/LW\right)\left(C(f)/f\right), \tag{1}$$

where L is the length of the conductor along the current flow, W is the width, and $C(f)$ is a weak function of the observation frequency f: $C(f) \propto 1/f^{(p-1)}$, where p is typically

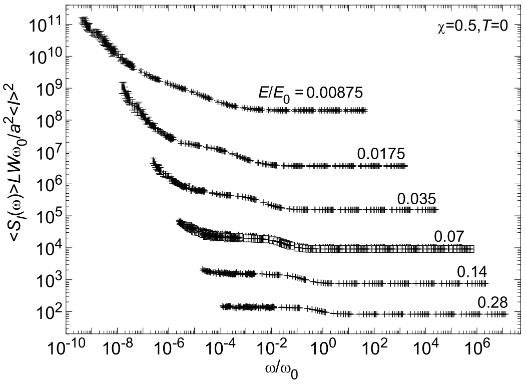

FIGURE 2. Spectral density $S_I(\omega)$ of current fluctuations at $T = 0$ and $\chi = 0.5$, normalized to the Hooge scaling factor $a^2 I^2/LW\omega_0$, as a function of observation frequency ω (measured in units of $\omega_0 \equiv g/\hbar v_0 a^2$) for several values of electric field. Each point represents data averaged over 48 samples.

between 1 and 2. Current spectral densities scaled in accordance with (1) are shown in Fig. 2 and seem to follow the $\langle I\rangle^2$ dependence nicely for different electric fields E and geometries. The power p is found to depend slightly on the electric field. For the smallest field accessible for numerical study the exponent p is close to 1.5, the value recently observed experimentally at 2D hopping in mesoscopic samples [10].

At moderately low frequencies, the noise spectral density as a function of f develops a plateau. This region of "broadband" noise may be crudely categorized according to which fundamental source of noise dominates, either thermal fluctuations or electron charge discreteness (shot noise). In the most interesting case of sufficiently low temperature, the thermal fluctuations are negligible, hence the broadband fluctuations are due almost entirely to the shot noise. In the case when the frequency dependence of the current spectral density $S_I(f)$ is flat, the strength of current fluctuations may be characterized by its ratio to the Schottky value $2eI$, i.e. using Fano factor $F \equiv S_I(f)/2eI$.

Recent experimental results [10, 11, 12] have shown that the shot noise may be indeed suppressed ($F < 1$). Theoretical and computational studies of shot noise at hopping in artificial space-ordered 1D [7, 13] and space-ordered and random 2D [14, 15, 16] systems have predicted that the shot noise suppression obeys

$$F = (L_c/L)^{\alpha}, \qquad L \gg L_c, \tag{2}$$

where L_c is a scaling constant which is interpreted as the average distance separating critical hops, which in percolation theory corresponds to a critical cluster size [5], and α is a positive exponent. Two examples of such scalings, for substantial ($\chi = 0.5$) and negligible ($\chi = 0$) Coulomb interactions, for different values E of the electric field, are shown in Fig. 3. The exponent α is found to be $\alpha = 1$ and $\alpha = 0.76 \pm 0.08$ for these two cases, respectively. Electric field dependences of L_c obtained for $\chi = 0.5$ and $\chi = 0$ using (2) are shown in Fig. 4. From Fig. 4, the shot noise suppression length $L_c(E)$

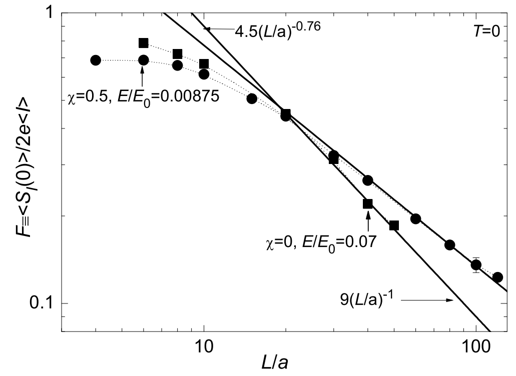

FIGURE 3. Average Fano factor F as functions of conductor length L for two values of applied field at $\chi = 0.5$, $T = 0$ and $W \gg L_c$.

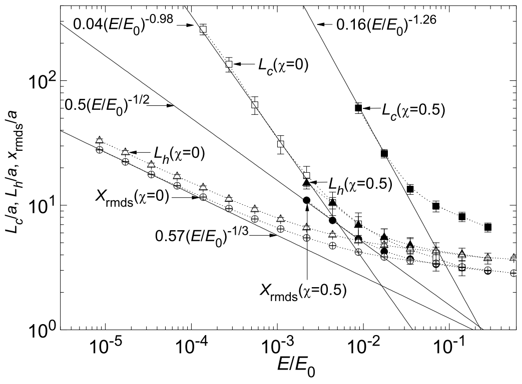

FIGURE 4. Cluster size L_c dependences on electric field E with ($\chi = 0.5$) and without ($\chi = 0$) the long-range Coulomb interaction included. Hop length x_{rmds} and high frequency scaling length L_h are also shown for comparison.

approaches the asymptote

$$L_c(E) \propto (E_0/E)^{\mu}\, a, \tag{3}$$

for sufficiently small E. The power μ of shot noise suppression is quite different from that determining the variable-range hopping length $x_{\mathrm{rmds}} \propto E^{-\beta}$, which is also shown in Fig. 4 for $\chi = 0$ ($\beta = 1/3$) and $\chi = 0.5$ ($\beta = 1/2$). Numerical values of the exponent μ are found to be $\mu = 0.98 \pm 0.08$ for zero Coulomb interaction strength $\chi = 0$ and $\mu = 1.26 \pm 0.15$ for $\chi = 0.5$, which are consistent with the estimates $\mu = \frac{1}{3}\left(1 + \nu_{\parallel}\right) \approx 0.91$ or $\mu = \frac{1}{2}\left(1 + \nu_{\parallel}\right) \approx 1.37$ obtained from the expression for the critical cluster size:

$$L_c \propto x_{\mathrm{rmds}} \left(x_{\mathrm{rmds}}/a\right)^{\nu_{\parallel}}, \tag{4}$$

where the critical cluster size index in the percolation direction $\nu_{\|}$ is 1.73 confirming the interpretation of L_c as the average directed percolation cluster size [17, 18, 19].

CONCLUSION

In summary, results of our numerical simulations demonstrate that the shot noise in single-electron arrays may substantially exceed its Schottky value $2eI$ due to remnants of electron-hole avalanches, which are only partly suppressed by the disorder.

On the other hand, shot noise at 2D hopping is suppressed below the Schottky value, in agreement with recent experiments. The characteristic length L_c determining this suppression is found to be in agreement with its interpretation as the average percolation cluster length.

ACKNOWLEDGMENTS

A major contribution from K. K. Likharev to the discussed results is greatfully acknowledged. This work was supported by the Department of Energy, by the Semiconductor Research Corporation, and by the Austrian Science Fund FWF, project P17285-N02.

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