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Green's function asymptotic in two-layered periodic medium

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Abstract. Green's function asymptotic properties of the wave field in a two-layered periodic medium is analytically examined. The solution is constructed by the stationary phase method. It is shown that some singularities occur in this approach. Namely, the Green's function asymptotical form is proportional to the distance from a source as $R^{-5/6}$, instead of the expected R^{-1} behavior. Thus, the power flux density in this directions decreases very slowly (as $R^{-5/3}$) and as a consequense there exists a set of ultra-propagation power channels. We focus on the description of the vicinity of a singular cone where the usual stationary phase method fails and determined a smooth transition from the standard asymptotic form to the new one. Finally, a detailed analysis of the obtained results is given.

Introduction

The Green's function method applied to periodic structure has theoretical significance, but studying propagation and scattering of waves is one of the most effective methods to investigate systems with periodically varying in space properties [1,2]. This method is widely adopted in investigations of liquid crystals [3] and photonic band structures [4]. Here we study the field of a point source, i.e. the Green's function in a two layered medium (a_1, a_2) with a step-like spatial variation of wave numbers (k_1, k_2) and one-dimensional periodicity for large distances from the source (Fig. 1). It is important to point out that the specified formulas do not depend on size and other structural parameters. It is shown that forbidden zones exist in such a media. The results are illustrated by numerical calculations.

1. Green's function integral representation

We consider a scalar field and are not interested in polarization effects. The Green's function $G(z, z_1, \mathbf{r}_\perp)$ is the solution of the equation

$$\left(\Delta + k^2(z)\right) G(z, z_1, \mathbf{r}_\perp) = -\delta(\mathbf{r} - \mathbf{r_1}),$$

which satisfies the limiting absorption principle. Taking advantage of the known integrated representation of the one-dimensional Green's function [5,6] we receive the following asymptotic representation

$$G(z, z_1, \mathbf{r}_\perp) \approx -\frac{1}{\sqrt{8\pi^3 r_\perp}} \int \frac{V(z_>)V^-(z_<)}{W(q)} \sqrt{q} \times \exp\left(i(qr_\perp + p(q)|z - z_1| - \pi/4)\right) dq. \quad (1)$$

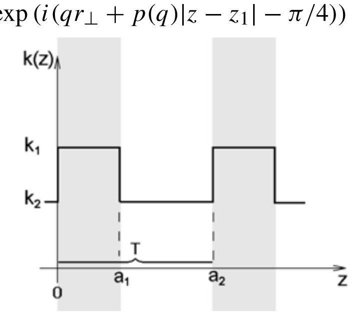

Fig. 1. Considered structure. The values a_1, a_2, k_1, k_2 are the layer widths and the wave numbers of the medium. T is the periodicity.

2. Method of a stationary phase

The asymptotic form of the Green's function in the far zone is to be considered, thus we calculate the integral (1) by the method of the stationary phase. In order to perform this it is necessary to find stationary points. The equation for definition of stationary points in polar coordinates (the origin is combined with a source $r_\perp = R\cos\varphi$, $|z - z_1| = R\sin\varphi$) looks like

$$p'(q) = -\text{ctg}\varphi. \quad (2)$$

A typical quasi-momentum derivative curve $p'(q)$ is represented in Fig. 2.

Let the polar angle vary in the sector $\varphi_* + \epsilon < \varphi < \pi/2$, where $\epsilon > 0$. Then the Green's function asymptotic is defined by the unique stationary point $q = q_1$ and can be presented in the form

$$G(z, z_1, \mathbf{r}_\perp, q_1) \approx -\frac{V(z_>)V^-(z_<)}{2\pi W(q_1)} \sqrt{\frac{q_1}{p''(q_1) r_\perp |z - z_1|}} \times \exp i\left(q_1 r_\perp + p(q_1)|z - z_1| + \pi/4(\text{sign}(p''(q_1)) - 1)\right). \quad (3)$$

3. Green's function singularities

Received asymptotic (3) becomes inapplicable in a vicinity of $q = q_*$, $p'(q_*) = -\text{ctg}\varphi_*$. At this point the function $p''(q)$ is zero (special direction). In order to construct a smooth asymptotic form we rewrite (3) as

$$G(z, z_1, \mathbf{r}_\perp, q_i) = A(z, z_1, r_\perp, q_i) \times \exp\left(iF(z, z_1, r_\perp, q_i)\right),$$

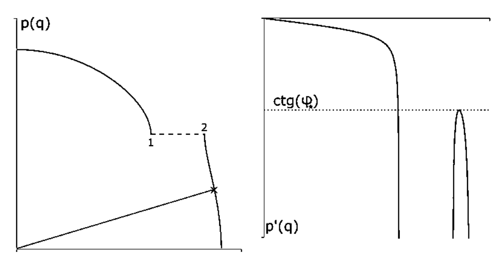

Fig. 2. The dispersion diagram $p(q)$ and quasi-momentum derivative curve $p'(q)$. The 1–2 dotted line shows the forbidden zone. It is visible, that the equation (2) has a unique root at $\text{ctg}\,\varphi < \text{ctg}\,\varphi_*$, two roots at $\text{ctg}\,\varphi = \text{ctg}\,\varphi_*$ and three roots in other cases. The angle φ_* is defined by the structure parameters.

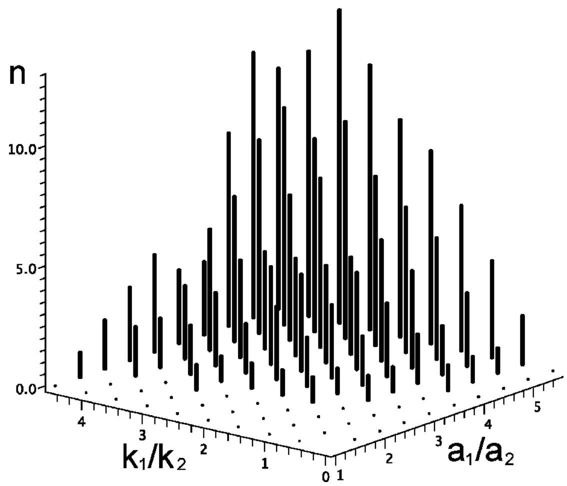

Fig. 3. The dependence of the forbidden zone number (n) on the relative layers width and the wave numbers. One can see that by varying the structure parameters in a certain way we obtain a different number of forbidden zones, i.e. number of special directions.

where $F(z, z_1, r_\perp, q_i) = q_i r_\perp + p(q_i)|z - z_1|$

$$A(z, z_1, r_\perp, q_i) = -\frac{e^{-i\frac{\pi}{4}}}{2\pi}\sqrt{\frac{q_i}{p''(q_i)r_\perp|z-z_1|}}\frac{V(z_>)V^-(z_<)}{W(q_i)}.$$

In this form the smooth asymptotic may be written as

$$G_s(z, z_1, \mathbf{r}_\perp, q_{(2,3)}) = e^{i\phi}\left(C_1 Ai(\psi) + C_2 Ai(1, \psi)\psi^{-1/2}\right),$$

where

$$\phi = (F(q_3) + F(q_2))/2,\ \psi = (3/4\,(F(q_3) - F(q_2)))^{2/3},$$
$$C_1 = \sqrt{\pi}\psi^{1/4}(A(q_3) + A(q_2)),$$
$$C_2 = i\sqrt{\pi}\psi^{1/4}(A(q_3) - A(q_2)).$$

Therefore, in the sector $0 < \varphi < \varphi_* + \epsilon$ the Green's function is the sum of two items

$$G(z, z_1, \mathbf{r}_\perp) = G(z, z_1, \mathbf{r}_\perp, q_1) + G_s\left(z, z_1, \mathbf{r}_\perp, q_{(2,3)}\right).$$

Here q_i are the roots of the equation (2). Similar expressions arise at the description of a wave field in the vicinity of a caustic. In our case this asymptotic form describes the complex interference picture in the far zone. It is possible to derive an expression for the power flux density if we define it as $P = i(G\nabla G^* - G^*\nabla G)/2$. In this case power propagation features the special directions, but proportional to $R^{-5/3}$ rather than R^{-2}. Results of numerical calculations of the Green's function and power flux density are represented in Fig. 4.

Thus, in contrast to a homogeneous medium, the Green's function for the periodic layered structure has a number of features. There are areas in which the wave field is described not only by a single wave, but by a sum of wave fields with the maximal number is defined by the properties of the structure and does not exceed number of extending normal waves for a wave guide consisting of two layers with periodic boundary conditions. Also, there exist allocated directions in which there is an occurrence or disappearance of an additional beam summand. The asymptotic of the wave field for these directions is described through the Airy function and in these directions the Green's function and the power flux density decreases slowly.

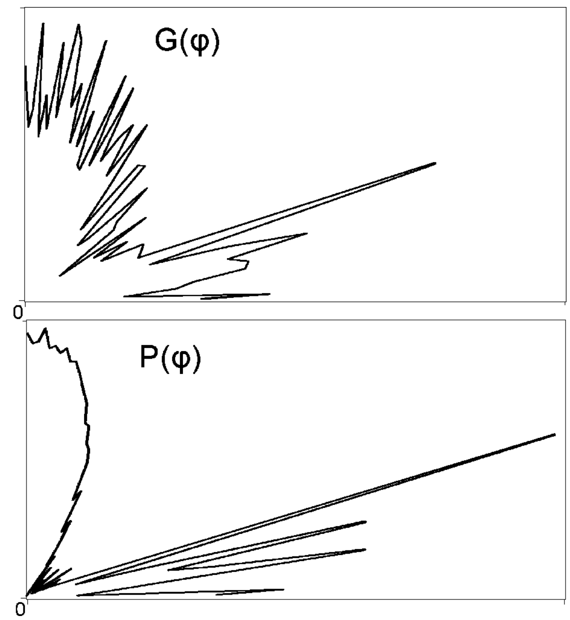

Fig. 4. Directional patterns of the Green's function $G(\varphi)$ and the power flux density $P(\varphi)$. The presence of the allocated directions corresponding to slower decrease are clearly visible.

Such a structure can be used as an attractive optical material for controlling and manipulating the light flow. The obtained results offer additional features leading to new device concepts (e.g. microscale structure for the lightconfinement with radically different characteristics compared to conventional optical fiber [7]), when some technological aspects such as manufacturability and principal difficulties such as disorder are being under control.

References

[1] M. S. Tomas, *Phys. Rev. A.* **51**, 2545 (1995).
[2] E. V. Aksenova, V. P. Romanov and A. Yu. Val'kov, *Phys. Rev. E.* **59**, 1184 (1999).
[3] V. A. Belyakov, *Diffraction Optics of Periodic Media with a Layered Structure* (Berlin: Springer-Verlag), 1992.
[4] K. M. Leung, *J. Opt. Soc. Am. B* **10**, 303 (1993).
[5] L. Brillouin and M. Parodi, *Propagation des Ondes dans les Milieux Périodiques* (Paris:Masson et cie editeurs,Dunodediteurs), 1956.
[6] L. Felsen and N. Marcuvitz, *Radiation and Scattering of waves* (New Jersey: Prentice Hall), 1973.
[7] J. D. Joannopoulos, S. Johnson, J. Winn and R. Meade, *Photonic Crystals: Molding the Flow of Light* (Princeton NJ: Princeton University), 2008.