# Novel Memristive Charge- and Flux-Based Sensors

Hiwa Mahmoudi, *Student Member, IEEE*, Viktor Sverdlov, and Siegfried Selberherr, *Fellow, IEEE*

Institute for Microelectronics, Technische Universität Wien, Gußhausstraße 27–29/E360, A-1040 Wien, Austria
E-mail: {mahmoudi|sverdlov|selberherr}@iue.tuwien.ac.at, Phone: +43-1-58801-36049, Fax: +43-1-58801-36099

***Abstract*—The fourth fundamental circuit element, memristor, is characterized by its electrical memory resistance (memristance), which is a function of the charge/flux. In this paper, using the unique ability of a memristor to record the historic profile of the applied current/voltage via its memristance, we propose novel memristive charge- and flux-based sensing schemes. Using a memristor, the capacitance, inductance, and power measurements are reduced to a resistance measurement. The proposed method seeks a memristive behavior with a constant modulation of the memristance (memductance) with respect to the charge (flux). We further demonstrate a charge-based capacitance sensor using a $TiO_2$ memristor. To have the possibility of both charge- and flux-based sensing, we suggest spintronic memristors based upon the current-driven dynamics of magnetic domain walls.**

***Index Terms*—Memristive sensing, domain wall-based memristor, spintronic**

## I. INTRODUCTION

The memristor was predicted by Chua based on a circuit theory symmetry argument [1]. However, it required four decades that, the first physical implementation of this missing circuit element was presented [2] based upon ionic transport ($TiO_2$ memristor). Shortly after that, spin-transfer torque (STT) memristors were proposed [3] based on the STT induced domain wall (DW) motion [4], [5].

The memristor is defined by a relationship between the flux ($\varphi(t) \equiv \int_{-\infty}^{t} v(\tau)\ d\tau$) and the charge ($q(t) \equiv \int_{-\infty}^{t} i(\tau)\ d\tau$), called constitutive relation. It acts as a programmable resistor which preserves its resistance, when the power is turned off. Thus, the most straightforward application of a memristor is a memory element. Memristive memory has many advantages over conventional one. It is nonvolatile, it does not have any leakage current, and it also enables "stateful" logic [6]. Since a memristor has a behavior similar to synapses, another potential application is to use them in neuromorphic systems [7], [8]. As the forth basic passive circuit element, a memristor is suited for analog circuit applications [9], [10].

The historic profile of the applied current or voltage memorized in the memristance change can be revealed instantaneously by measuring its varying resistance. This storage capability, which is independent of the memristor material, can reduce the charge (flux)-based capacitance (inductance) sensing to a simple resistance measurement, as we show below. The proposed sensing scheme is also suited for power management [11]. First, we present charge-based capacitance sensing using a $TiO_2$ memristor. Next, using domain wall-based spintronic memristors [3], we demonstrate that both capacitance and inductance can be addressed.

## II. SENSING PRINCIPLES

The fundamental circuit variables are electrical current $i$, voltage $v$, charge $q$, and magnetic flux $\varphi$. Basic circuit elements (resistor $R$, capacitor $C$, inductor $L$, and memristor $M$) are defined in terms of the relationships between two of these variables as $R = dv/di$, $C = dq/dv$, $L = d\varphi/di$, $M = d\varphi/dq$. The behavior of basic electrical circuits are determined by Kirchhoff's current (KCL) and voltage (KVL) laws. A resistor relates the voltage and the current by a linear relationship called Ohm's law. Its resistance, therefore, can be determined instantaneously by measuring the current and the voltage simultaneously. A capacitor and an inductor, however, relate the voltage to the current through differential equations, thus, RLC-based capacitance and inductance measurement methods are therefore entirely different from those used for the resistance. Memristance, however, can be determined instantaneously. Below we demonstrate how by using a charge (flux)-controlled memristor one can efficiently determine capacitance (inductance) and perform power measurement.

The constitutive relations of a charge-controlled [12] memristor is expressed as $\varphi = \varphi(q)$ and its memristance is defined as $M(q) = d\varphi/dq$. Therefore:

$$\frac{dM(q)}{dt} = \frac{dM(q)}{dq}\frac{dq}{dt} = \frac{dM(q)}{dq} i_{\mathrm{M}} \tag{1}$$

According to the capacitance definition we have:

$$C = \frac{dq}{dv} = \frac{dq/dt}{dv_{\mathrm{C}}/dt} = \frac{i_{\mathrm{C}}}{dv_{\mathrm{C}}/dt} \tag{2}$$

Let us consider a charge-controlled memristor connected in series with a capacitor ($i_{\mathrm{M}} = i_{\mathrm{C}}$). By substituting $i_{\mathrm{M}}$ from (1) into (2) we obtain:

$$C = \left(\frac{dM(q)}{dq}\right)^{-1} \frac{dM/dt}{dv_{\mathrm{C}}/dt} = \left(\frac{dM(q)}{dq}\right)^{-1} \frac{dM}{dv_{\mathrm{C}}} \tag{3}$$

The term $dM/dq$ is related to the intrinsic properties of the memristor. For a linear resistor this term is zero. The charge-controlled memristor with this term being constant is suited for charge-based capacitance and power measurement. The capacitance is determined by measuring changes in the memristance and the voltage across the capacitor.

We now describe the capacitance sensing scheme by using a $TiO_2$ memristor. The $TiO_2$ memristor (shown in the bottom inset of Fig. 1) acts as two resistors in series. The doped region of the length $w$ ($TiO_{2-x}$) has the resistance $R_{\mathrm{ON}}w/T$, while the undoped region ($TiO_2$) has the resistance $R_{\mathrm{OFF}}(1-w/T)$, where $R_{\mathrm{ON}}$ ($R_{\mathrm{OFF}}$) is the minimum (maximum) resistance of the device, when it is completely doped (undoped), and $T$ denotes the thickness of the semiconductor film. The current flowing through the memristor changes the size of the doped region, thus altering the effective resistance of the memristor.

The memristance is obtained as:

$$M(q) = R_{\mathrm{OFF}}\left[1 - \frac{\mu_{\mathrm{V}} R_{\mathrm{ON}}}{T^2} q(t)\right] \tag{4}$$

$\mu_{\mathrm{V}}$ is the mobility of dopants [2]. The term $dM/dq$ is a constant equal to $-1.44 \times 10^9 \Omega \mathrm{C}^{-1}$ for a typical $\mathrm{TiO_2}$ memristor with $R_{\mathrm{ON}} = 100\Omega$, $R_{\mathrm{OFF}}/R_{\mathrm{ON}} = 360$, $\mu_{\mathrm{V}} = 10^{-10}\mathrm{cm}^2\mathrm{s}^{-1}\mathrm{V}^{-1}$, and $T = 5\mathrm{nm}$ [2]. Fig. 1 shows the I-V characteristics of the $\mathrm{TiO_2}$ memristor with applied ac voltage $v_s(t) = sin(wt)$ at $w_1 = 40\mathrm{rad/s}$ and $w_2 = 20\mathrm{rad/s}$. The memristor behaves like a nonlinear resistor which shows a wide variety of IV characteristics depending on the frequency.

Let us consider a memristor-capacitor (MC) circuit shown in the inset of Fig. 2(d). Fig. 2(a) shows the I-V characteristics of the memristor, when the MC circuit is excited by a step voltage source $v_{\mathrm{s}}$. The characteristics display different behavior for different capacitances. Using the time dependent voltage across the capacitor (Fig. 2(b)) and the memristance (Fig. 2(c)), one can determine the capacitance according to (3), at a time interval much shorter than the charging time constant (Fig. 2(d)).

For a flux-controlled [12] memristor the constitutive relation is expressed as $q = q(\varphi)$ and its memductance is defined as $G(\varphi) = dq/d\varphi$. So we have:

$$\frac{dG(\varphi)}{dt} = \frac{dG(\varphi)}{d\varphi}\frac{d\varphi}{dt} = \frac{dG(\varphi)}{d\varphi} v_{\mathrm{M}} \tag{5}$$

Similar to (3), it can be shown that when a flux-controlled memristor is connected in parallel to an inductor ($v_{\mathrm{M}} = v_{\mathrm{L}}$), the inductance is obtained as:

$$L = \left(\frac{dG(\varphi)}{d\varphi}\right)^{-1} \frac{dG/dt}{di_{\mathrm{L}}/dt} = \left(\frac{dG(\varphi)}{d\varphi}\right)^{-1} \frac{dG}{di_{\mathrm{L}}} \tag{6}$$

A flux-controlled memristor with the term $dG/d\varphi$ being constant is thus suited for flux-based inductance measurement. In the following sections we consider the possible application of this memristive behavior in power monitoring.

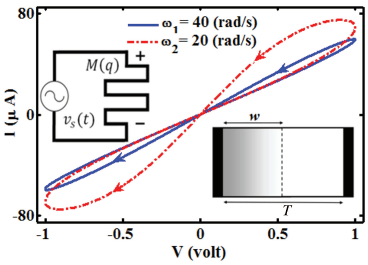

Fig. 1. The memristor symbol and the applied voltage source (top inset), the memristance of the $\mathrm{TiO_2}$ memristor structure (bottom inset) [2], and I-V characteristics plotted for the memristor with the initial state $w/T = 0.5$, at two different frequencies of ac voltage.

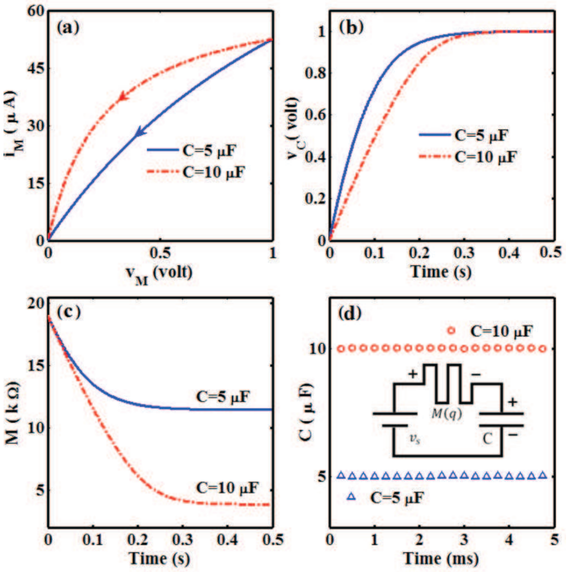

Fig. 2. (a) $\mathrm{TiO_2}$ memristor I-V curves. (b) The voltage of the capacitor. (c) The memristance. (d) The related electric circuit (inset) and determined capacitances using (3).

## III. Power monitoring

The unique ability of a memristor to record the historic profile of the voltage/current applied makes it suitable for power measurement [11]. The total energy generated by an electric power supply $E = \int V_s(t)I_s(t)dt$, where $V_s(t)$ is the voltage across the source and $I_s(t)$ is the current flowing through it. For a circuit powered by a constant voltage source (Fig. 3(a)) the energy is given by:

$$E = V_s \int I_s(t)dt = V_s \Delta q = V_s \Delta M \left(\frac{dM(q)}{dq}\right)^{-1} \tag{7}$$

Therefore, a charge-controlled memristor with a constant term $dM(q)/dq$ reduces the power measurement to the memristance measurement.

In a circuit powered by a constant current source (Fig. 3(b)) a flux-controlled memristor with the term $dG(\varphi)/d\varphi$ being constant reduces the power measurement to the memductance measurement:

$$E = I_s \int V_s(t)dt = I_s \Delta\varphi = I_s \Delta G \left(\frac{dG(\varphi)}{d\varphi}\right)^{-1} \tag{8}$$

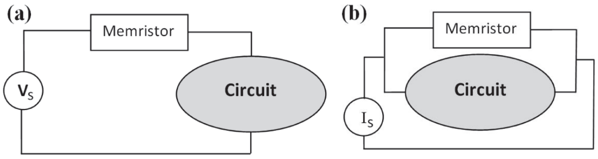

Fig. 3. Memristive power monitoring circuits [11]. Memristive charge-based (a) and flux-based (b) measurements reduce the energy measurement to a resistance measurement. In (a)/(b), the memristance/memductance has to be much smaller than the circuit resistance/conductance to minimize the impact of the memristor on measurements.

In the following section we study different spintronic memristors which are suited for both charge- and flux-based measurements.

## IV. SPINTRONIC MEMRISTIVE SENSING

A thin-film element with varying width $w$ and constant thickness $t$ (Fig. 4a) has been proposed as a spintronic memristor [3] based upon the STT induced DW motion in the film-length direction $x$. The dynamic properties of a propagating magnetic DW are strongly affected by the device geometry [15]. STT memristors exhibit therefore a geometry dependent memristive behavior. The constitutive relation is $\varphi(q) = Aq^{(1-\rho k)/(\rho+1)}$ [3], where A is a constant coefficient, and $k$ defines the DW mobility which is scaling with the aspect ratio $\mu \sim (w/t)^k$. $\rho$ determines the spatial dependence of $w$ as $w = w(x) = \tilde{w}\,(x/\tilde{x})^\rho$, where $\tilde{w}$ represents the value of $w$ at a particular location $\tilde{x}$. It can be shown that, for spatial shapes with $\rho = -1/(k+2)$ (Fig. 4b) and $\rho = 1/(2k+1)$ (Fig. 4c), the terms $dM/dq$ and $dG/d\varphi$ are constant and therefore, the memristors can be used for charge- and flux-based measurements, respectively.

Two other nanoscale spintronic devices with memristive behavior [3] are shown in Fig. 5. A spin-valve memristor consists of two ferromagnetic layers as shown in Fig. 5a. One layer has a pinned magnetization direction (pinned layer) and the other is divided by a DW into two segments with opposite magnetization directions. The electrical resistance of the device is a function of the DW position $(X)$ [3] as:

$$M(x) = \frac{v_\mathrm{M}}{i_\mathrm{M}} = R_{\uparrow\uparrow} + R_{\uparrow\downarrow} = R_\mathrm{H} - x(R_\mathrm{H} - R_\mathrm{L}) = (1 + x\mathrm{GMR})R_\mathrm{L} \qquad (9)$$

$x = X/D$ and $D$ represent the free layer length. $R_\mathrm{L}$ and $R_\mathrm{H} = (1 + \mathrm{GMR})R_\mathrm{L}$ are the low and high resistances of the device, respectively. GMR is the giant magnetoresistance ratio. As shown in Fig. 5b, the memductance of an MTJ-based memristor can be expressed as the sum of two conductances [14] as:

$$G(x) = \frac{i_\mathrm{M}}{v_\mathrm{M}} = G_{\uparrow\uparrow} + G_{\uparrow\downarrow} = xG_\mathrm{H} + (1-x)G_\mathrm{L} = (1 + x\mathrm{TMR})G_\mathrm{L} \qquad (10)$$

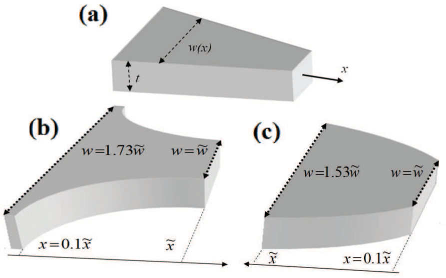

Fig. 4. (a) Thin-film spintronic memristor [3]. Proposed structures with constant parameters of $dM/dq$ (a) and $dG/d\varphi$ (b). The parameter $k$ is supposed to be equal to 2.2 [13].

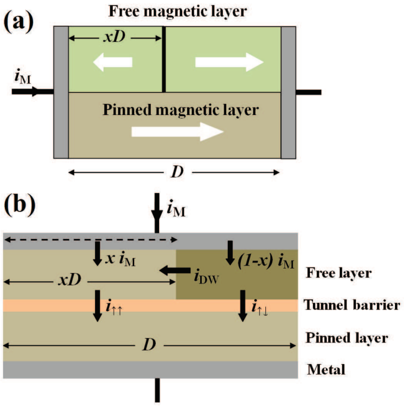

Fig. 5. Spin-valve (a) and MTJ-based (b) spintronic memristors [3], [14].

$G_\mathrm{L}$ and $G_\mathrm{H} = (1+\mathrm{TMR})G_\mathrm{L}$ are the low and high conductance, respectively. TMR is the tunneling magnetoresistance ratio.

We obtain the term $dM/dq$ and $dG/d\varphi$ for the spin-valve and the MTJ-based memristors, respectively as:

$$\frac{dM}{dq} = \frac{dM}{dx}\frac{dx}{dq} = R_\mathrm{L}\mathrm{GMR}\frac{dx/dt}{dq/dt} = R_\mathrm{L}\mathrm{GMR}\frac{V_\mathrm{DW}}{i_\mathrm{M}} \qquad (11)$$

$$\frac{dG}{d\varphi} = \frac{dG}{dx}\frac{dx}{d\varphi} = G_\mathrm{L}\mathrm{TMR}\frac{dx/dt}{d\varphi/dt} = G_\mathrm{L}\mathrm{TMR}\frac{V_\mathrm{DW}}{v_\mathrm{M}} \qquad (12)$$

$V_\mathrm{DW}$ is the DW velocity. According to the last two equations, the spin-valve (MTJ-based) memristor is suited for charge-(flux)-based measurement, when the relationship between $V_\mathrm{DW}$ and $i_\mathrm{M}$ ($v_\mathrm{M}$) is linear. In the absence of an external field and extrinsic pinning, a linear dependence of the DW velocity with respect to the applied drive is expected, when the ratio $\alpha/\beta$ is nonzero, where $\alpha$ is the damping parameter and $\beta$ defines the strength of the non-adiabatic spin-torque [16]. Thus, in the presence of non-adiabatic effects, spin-valve and MTJ-based memristors are suited for charge- and flux-based sensors.

For a design example of a spintronic capacitance sensor, the average DW velocity as a function of the current passing through the spin-valve memristor is shown Fig. 6a, based on the one-dimensional model of DW dynamics [17]. According to (11), the memristor is suited for capacitance sensing when $\beta \neq 0$. As shown in Fig. 6b, the sensitivity increases with the ratio $\beta/\alpha$, but at the cost of decreased sensing range. In fact, the increase in the ratio $\beta/\alpha$, increases the DW velocity and therefore also the sensitivity.

## V. READOUT METHODS AND APPLICATIONS

Since the memristor holds the information, it is possible to use the memristor in a readout circuit which measures the memristance and also resets the memristor for the next measurement. Therefore, unlike other time domain methods, memristive sensing does not need an extra hardware for time/frequency measurement. In fact, a memristive sensor can be simply implemented using a switch. This implementation needs a low power hardware platform so that the sensor is low

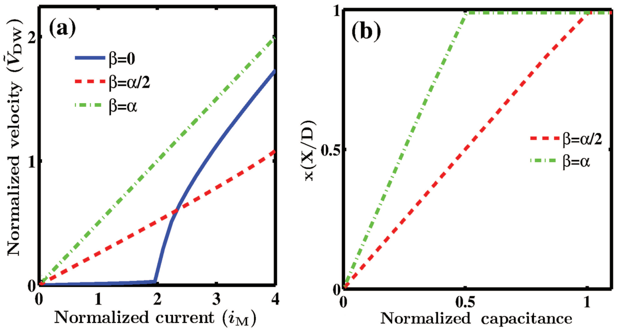

Fig. 6. (a) The average DW velocity ($V_{\mathrm{DW}}$) in the spin-valve structure as a function of the memristor current $i_{\mathrm{M}}$. (b) The final value of the variable $x$ as a function of the capacitance plotted for the spin-valve memristor in an MC circuit excited by a step voltage. The final memristance is different for different capacitances and its sensitivity increases with $\beta$ at the cost of decreased sensing range.

power and inexpensive. Fig. 7a and Fig. 7b show the readout circuits [11], which can determine the metastasis to have an analog sensor output. This can be used for applications like power management and capacitive position, humidity, fluid level, acceleration, etc. The readout circuit shown in Fig. 7c acts as a comparator to provide a digital output, which can be used as a capacitance (inductance) to digital converter for applications in capacitive touch sensing.

## VI. CONCLUSION

In this paper, novel charge- and flux-based sensing schemes are proposed based on the unique property of memristors to memorize the charge (flux) which passed through the device. The proposed methods can be used for capacitance, inductance, and power measurements. Although inductance and capacitance sensing are far from being new problems, the use of a memristor reduces the measurement to a straightforward resistance measurement which can be performed fast. We have shown that the $TiO_2$ memristor can be used for charge-based measurements. Spintronic memristors are proposed for both charge- and flux-based capacitance and inductance measurement. Our sensing method is suitable for measuring time-varying inductances and capacitances and it has the potential to be used in novel inductive and capacitive sensors. Proposed memristive sensing schemes are also suitable for power monitoring. In any case, memristors reduce the problem to a straightforward resistance measurement.

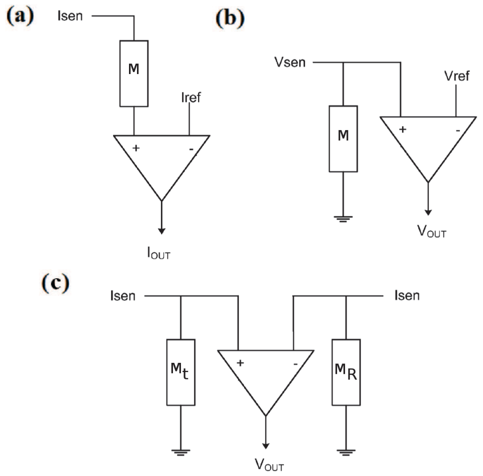

Fig. 7. Readout circuits. In (a) and (b), the value of the memristance is measured [11] to determine the value of the sensed capacitance, inductance or power. In (c) the value of the memristance is compared to a reference value to have a digital output. $M_{\mathrm{R}}$ and $M_{\mathrm{t}}$ are the reference and test memristors which are used for sensing a reference and a target capacitance (inductance), respectively.

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