J Supercond Nov Magn (2013) 26:1745–1748
DOI 10.1007/s10948-012-2034-y

ORIGINAL PAPER

Influence of Geometry on the Memristive Behavior of the Domain Wall Spintronic Memristors and Its Applications for Measurement

H. Mahmoudi · V. Sverdlov · S. Selberherr

Received: 4 November 2012 / Accepted: 1 December 2012 / Published online: 28 December 2012

Abstract A memristor is characterized by its electrical memory resistance (memristance), which is a function of the historic profile of the applied current (voltage). This unique ability allows reducing charge- and flux-based measurements to straightforward resistance measurements. The memristive measurement seeks a memristor with a constant modulation of the memristance (memductance) with respect to the charge (flux) for charge (flux)-based measurements. In this work the geometry dependent memristive behavior of a spintronic device is studied to demonstrate the possibility of both charge- and flux-based sensing, using spintronic memristors with different device geometries. The dynamic properties of a propagating magnetic domain wall in different geometrical structures make the spintronic memristor suitable for the charge-based capacitance and flux-based inductance measurements.

Keywords Magnetic domain wall · Memristive measurement · Spintronic memristor

1 Introduction

The memristor, or memory resistor, is the fourth fundamental circuit element which is defined by a functional relationship between two circuit variables, charge q and flux φ (the time integral of voltage), called constitutive relation. The memristor was predicted based on a symmetry argument of circuit theory in 1971 [1]. About four decades later, the physical realization of the first memristor based on ionic transport in TiO_{2-x} films [2] attracted much interest worldwide. Shortly after that, spintronic memristors were proposed [3] based on the spin-transfer torque (STT) effect. The STT mechanism gives memristive capabilities to a spintronic device for which the total electrical resistance depends upon the magnetization state which itself is a function of the cumulative effects of electron spin propagations.

The magnetization state of a thin-film element (Fig. 1a) including a magnetic domain wall (DW) manipulated by the electric current is determined by the historic profile of the applied current (voltage). Having a varying width, the device exhibits a memristive behavior, while the total electrical resistance is a function of the DW position [3]. The dynamic properties of the propagating DWs are strongly affected by the device geometry [4, 5]. Therefore, the memristive behavior of the device is a function of the geometrical structure of the memristor. Here, the rich dynamic behavior of spintronic memristors is studied to address both charge- and flux-based memristive measurements.

2 Capacitance and Inductance Memristive Measurement

The basic circuit elements (resistor R, capacitor C, inductor L, and memristor M) are defined in terms of functional relationships between two of four circuit variables (current i, voltage v, charge q, and flux φ) as $R = dv/di, C = dq/dv, L = d\varphi/di$, and $M = d\varphi/dq$. Memristors act as programmable (nonlinear) resistors and have the same unit as resistors (Ω) in contrast to capacitors (F) and inductors

H. Mahmoudi (✉) · V. Sverdlov · S. Selberherr
Institute for Microelectronics, Technische Universität Wien,
Gußhausstraße 27-29, 1040 Wien, Austria
e-mail: mahmoudi@iue.tuwien.ac.at

V. Sverdlov
e-mail: sverdlov@iue.tuwien.ac.at

S. Selberherr
e-mail: selberherr@iue.tuwien.ac.at

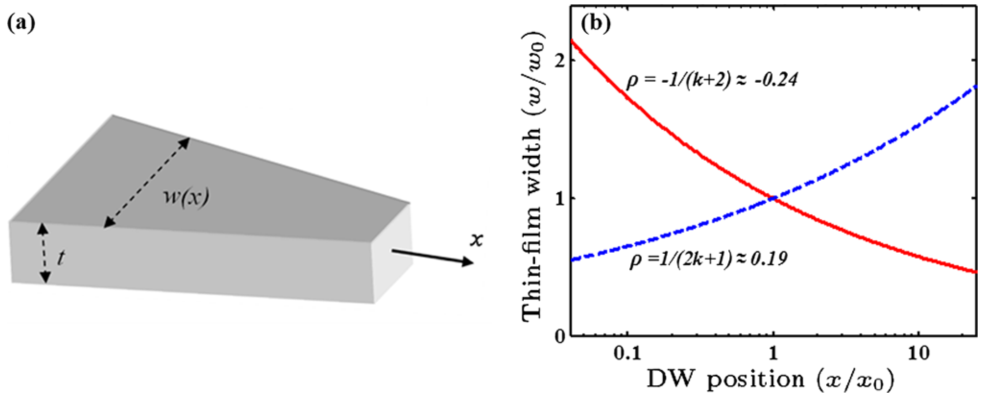

Fig. 1 (**a**) Domain wall spintronic memristor [3]. (**b**) Desired device geometries for charge-based (*solid line*) and flux-based (*dashed line*) memristive sensing

(H). The memristance, therefore, can be determined instantaneously by measuring the current and the voltage simultaneously. The behavior of basic electrical circuits is determined by Kirchhoff's current (KCL) and voltage (KVL) laws. As capacitors and inductors relate the voltage to the current through differential equations, the RLC-based capacitance and inductance measurement methods are entirely different from those used for resistance and memristance measurement. The unique ability of the memristor to memorize the history of the applied current or voltage instantaneously leads to a sensing capability which cannot be achieved by RLC-networks alone. In [6] we have shown that, regardless of the memristor device material and operating mechanism, when a charge (flux)-controlled memristor is connected in series (parallel) with a capacitor (inductor), the capacitance (inductance) is determined as

$$C = \frac{\Delta q}{\Delta v_C} = \left(\frac{dM(q)}{dq}\right)^{-1} \frac{\Delta M(q)}{\Delta v_C} \tag{1a}$$

$$L = \frac{\Delta \varphi}{\Delta i_L} = \left(\frac{dG(\varphi)}{d\varphi}\right)^{-1} \frac{\Delta G(\varphi)}{\Delta i_L} \tag{1b}$$

where v_C (i_L) is the voltage (current) of the capacitor (inductor) and M (G) is the memristance (memductance). The term dM/dq ($dG/d\varphi$) is related to the intrinsic properties of the memristor. These terms are zero for a conventional (linear) resistor. A charge (flux)-controlled memristor with this term being constant is suited for capacitance (inductance) measurements. In the following section we study the geometry dependent dynamic behavior of the DW spintronic memristor to find a proper device structure for memristive measurements.

3 Geometry Dependent Memristive Behavior

The current-induced DW motion in a magnetic thin-film element exhibits memristive properties, when the thin-film aspect ratio (w/t) [4] is varying with the film-length direction x. For the device with constant thickness t, the mobility of the DW, thus the electrical resistance (memristance M) of the device, is a function of the thin-film element width (w) and is expressed as [3]

$$M(w) = M_0 \left(\frac{w}{w_0}\right)^{-(k+1)} \tag{2}$$

Here M_0 and w_0 are the resistance and the width of the element, respectively, when the DW is located at x_0 and k defines the DW mobility scaling with the aspect ratio $\mu \sim (w/t)^k$. When the spatial dependence of the element width as a function of the DW position is given by $w(x) = w_0(x/x_0)^\rho$, the memristance is determined as [3]

$$M(x) = M_0 \left(\frac{x}{x_0}\right)^{-\rho(k+1)} \tag{3}$$

To determine the memristive behavior of the device, which is significantly affected by the DW dynamic properties, the DW velocity (dx/dt) is assumed to be proportional to the current density [3]. Therefore the memristor constitutive relation is given by

$$\varphi(q) = Aq^{(1-\rho k)/(\rho+1)} \tag{4}$$

where A is a constant coefficient. According to (1a), (1b), the suited device geometries for charge- and flux-based sensing are determined as (5a) and (5b), respectively. We have

$$\frac{dM(q)}{dq} \equiv \frac{d^2\varphi(q)}{dq^2} = \text{const.} \quad \Rightarrow \quad \rho = -\frac{1}{k+2} \tag{5a}$$

$$\frac{dG(\varphi)}{d\varphi} \equiv \frac{d^2q(\varphi)}{d\varphi^2} = \text{const.} \quad \Rightarrow \quad \rho = \frac{1}{2k+1} \tag{5b}$$

The desired structure geometries are shown in Fig. 1b for $k = 2.2$ [4].

In order to taking a closer look at the DW dynamics in a patterned structure, we use the one-dimensional model of the DW dynamics [7–10]. The dynamics of the DW position x and position-dependent magnetization $\boldsymbol{S} = S(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ is described as

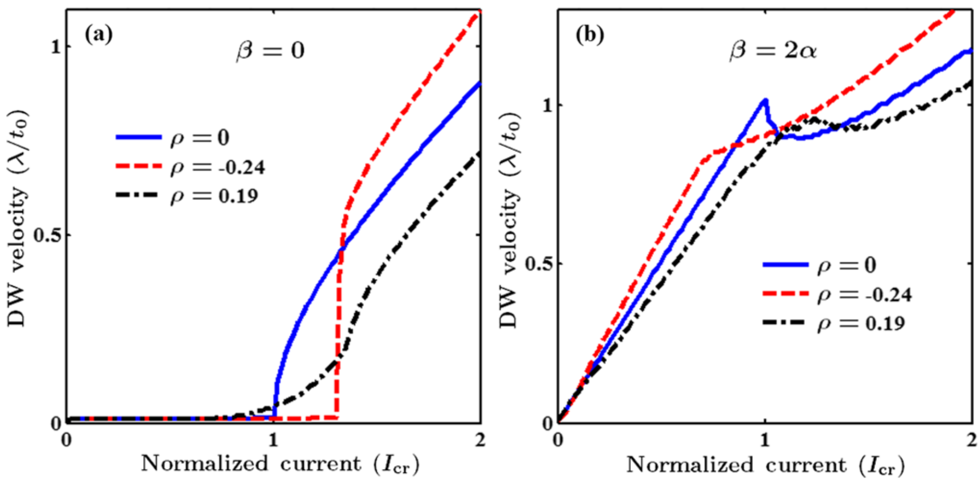

Fig. 2 Time-averaged domain wall velocity in the absence (**a**) and presence (**b**) of the non-adiabatic spin-torque effect plotted for different geometrical structures (ρ). When β is nonzero (**b**), the DW propagation is characterized by a linear regime and a turbulent regime above the Walker breakdown. With different geometries (ρ) the device has different values of the Walker breakdown current. The *axes* are dimensionless, with position x, time t, and current I, expressed in units of λ, $t_0 = 2\hbar/SK_T$, and I_{cr}

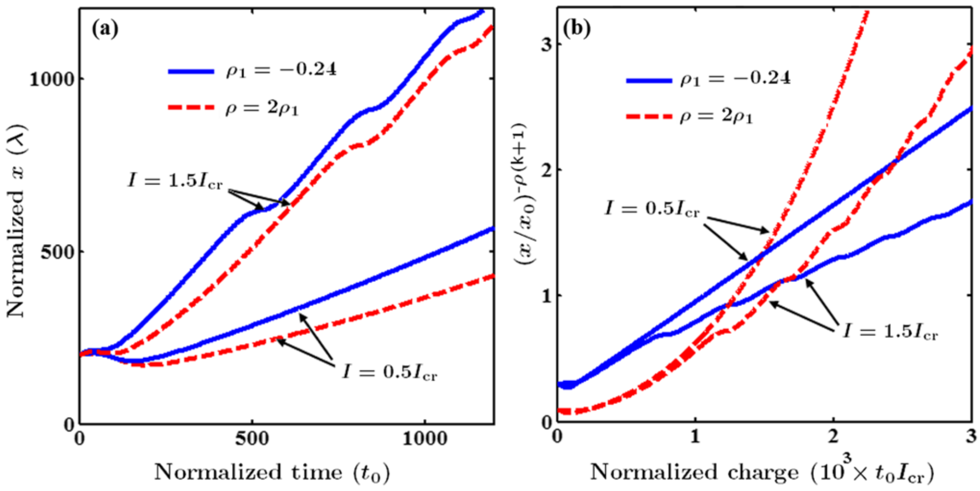

Fig. 3 Dynamics of a current-driven domain wall in a thin-film element with a varying width. A DW position as a function of time (**a**) and the modulation of the term $(x/x_0)^{-\rho(k+1)}$ (normalized memristance) with the charge flowing through the device (**b**) plotted for two different geometries (ρ and ρ_1)

$$\lambda\frac{d\varphi}{dt} + \alpha\frac{dx}{dt} = \beta v_{\mathrm{el}} \tag{6a}$$

$$\frac{dx}{dt} - \lambda\alpha\frac{d\varphi}{dt} = \frac{SK_T}{2\hbar}\lambda\sin(2\varphi) + v_{\mathrm{el}} \tag{6b}$$

where φ is the angle between spins at the wall center and the easy plane, λ is the DW thickness, α is the damping parameter, K_L and K_T are the longitudinal and transverse anisotropy constants, and β defines the strength of the non-adiabatic spin torque. v_{el} is the spin-torque excitation strength determined as

$$v_{el} = \frac{\lambda\eta}{eNS}I_{DW} \tag{7}$$

where η denotes the polarization efficiency, e is the electron charge, and I_{DW} is the current following through the DW cross-sectional surface (wt). N is the number of spins in the DW and a denotes the lattice constant. To take into account the varying width of the element, $w(x)$, we obtain the memristor dynamics by solving numerically the coupled equations (3), (6a), (6b), and (7), for which the number of spins in the DW is a function of the DW position ($N(x) = 2w(x)t\lambda/a^3$).

Figure 2 shows the geometry dependence of the DW dynamics for zero and nonzero non-adiabatic spin-torque values. In the absence of the non-adiabatic term (Fig. 2a), there is a geometry dependent threshold current required to move a DW. Therefore, for the currents below a (geometry dependent) current threshold the memristive behavior cannot be observed. In the presence of the non-adiabatic term (Fig. 2b), the average DW velocity increases linearly with the applied current for low current values. Therefore, with $\beta \neq 0$, the memristive behavior can be observed and with proper geometry, the device can be used for memristive sensing. Above a (geometry dependent) critical current value called the Walker breakdown [11, 12], the DW motion becomes non-uniform and shows complex behavior as demonstrated in Fig. 3.

For two different geometrical structures and current levels, the dynamics of the DW position (x) are shown in Fig. 3a. With the desired geometry ($\rho = -0.24$), the term

$(x/x_0)^{-\rho(k+1)}$ has a constant modulation with charge for the current levels below a critical value as shown in Fig. 3b. Therefore, according to (3), the device is suited for charge-based measurement. This memristive sensor shows nonlinear behavior at high current values, which results from the Walker breakdown. Similarly, by using the one-dimensional model of the DW dynamics with varying number of spins in the DW, it can be shown that at the current/voltage levels below a critical value, the memductance ($M^{-1}(x) \sim (x/x_0)^{\rho(k+1)}$) has a constant modulation with flux for a geometrical structure with $\rho = 0.19$. This geometry, therefore, is suited for flux-based measurement.

4 Sensitivity

The sensitivity of the memristive charge/flux-based sensing scheme is determined by the value of the memristance/memductance modulation with respect to the charge/flux of the memristor. We define ΔQ as the amount of charge passed through the memristor which changes the memristance from its minimum to its maximum value. For TiO_2 memristors [2], since the mobility of dopants (oxygen vacancies) in titanium dioxide is low ($\sim 10^{-10}\ \mathrm{cm}^2\,\mathrm{s}^{-1}\,\mathrm{V}^{-1}$ [2]), ΔQ is in the range of tens to hundreds of micro-coulombs for a nanometer-scale motion of the doping front. It turns out that it is in the range of nano-coulombs to pico-coulombs in spintronic memristors for a micrometer-scale motion of the magnetic domain wall (extracted from experimental results of [4] and [13]). Our results indicate that the memristive capacitance sensors based on TiO_2 memristors can measure capacitances in the range of micro-farads to nano-farads [6]. Since spintronic memristors are more sensitive and can be more finely tuned, they are promising for measuring capacitances of 3–6 orders of magnitude lower than that measured by the TiO_2 memristors.

5 Conclusion

The effect of the device geometry on the memristive behavior of a spintronic device is studied to determine proper geometries for memristive charge- and flux-based sensing. In the presence of the non-adiabatic spin-torque effect, the device shows memristive behavior at low current/voltage regimes and within the desired geometries the device has a constant modulation of the memristance (memductance) with respect to the charge (flux) applied, which can be used for capacitance (inductance) measurement. Although inductance and capacitance sensing are far from being new problems, the use of a memristor reduces the measurement to a straightforward resistance measurement which can be performed fast. The memristive sensing method is suitable for measuring time-varying inductances and capacitances and it has the potential to be used in novel inductive and capacitive sensors.

Acknowledgement The work is supported by the European Research Council through the grant #247056 MOSILSPIN.

References

1. Chua, L.O.: Memristor – the missing circuit element. IEEE Trans. Circuit Theory **18**, 507–519 (1971)
2. Strukov, D.B., Snider, G.S., Stewart, D.R., Williams, R.S.: The missing memristor found. Nature **453**, 80–83 (2008)
3. Wang, X., Chen, Y., Xi, H., Li, H., Dimitrov, D.: Spintronic memristor through spin torque induced magnetization motion. IEEE Electron Device Lett. **30**, 294–297 (2009)
4. Berger, L.: Analysis of measured transport properties of domain walls in magnetic nanowires and films. Phys. Rev. B **73**, 014407 (2006)
5. Wunderlich, J., Ravelosona, D., Chappert, C., Cayssol, F., Mathet, V., Ferre, J., Jamet, j.-P., Thiaville, A.: Influence of geometry on domain wall propagation in a mesoscopic wire. IEEE Trans. Magn. **37**, 2104–2107 (2001)
6. Mahmoudi, H., Sverdlov, V., Selberherr, S.: Novel memristive charge- and flux-based sensors. In: Proceedings of the 8th Conference on Ph.D. Research in Microelectronics & Electronics (PRIME), pp. 15–18 (2012)
7. Tatara, G., Kohno, H.: Theory of current-driven domain wall motion: spin transfer versus momentum transfer. Phys. Rev. Lett. **92**, 086601 (2004)
8. Thomas, L., Parkin, P.: Current induced domain-wall motion in magnetic nanowires. In: Kronmüller, H., Parkin, S. (eds.) Handbook of Magnetism and Advanced Magnetic Materials, pp. 942–982. Wiley, Chichester (2007)
9. Duine, R.A., Nunez, A.S., MacDonald, A.H.: Thermally assisted current-driven domain-wall motion. Phys. Rev. Lett. **98**, 056605 (2007)
10. Jung, S.W., Kim, W., Lee, T.D., Lee, K.J., Lee, H.W.: Current-induced domain wall motion in a nanowire with perpendicular magnetic anisotropy. Appl. Phys. Lett. **92**, 202508 (2008)
11. Schryer, N.L., Walker, L.R.: The motion of 180° domain wall in uniform dc magnetic field. J. Appl. Phys. **45**, 5406 (1974)
12. Mougin, A., Cormier, M., Adam, J.P., Metaxas, P.J., Ferré, J.: Domain wall mobility, stability and walker breakdown in magnetic nanowires. Europhys. Lett. **78**, 57007 (2007)
13. Yamaguchi, A., Ono, T., Nasu, S., Miyake, K., Mibu, K., Shinjo, T.: Erratum: real-space observation of current-driven domain wall motion in submicron magnetic wires. Phys. Rev. Lett. **92**, 077205 (2004)